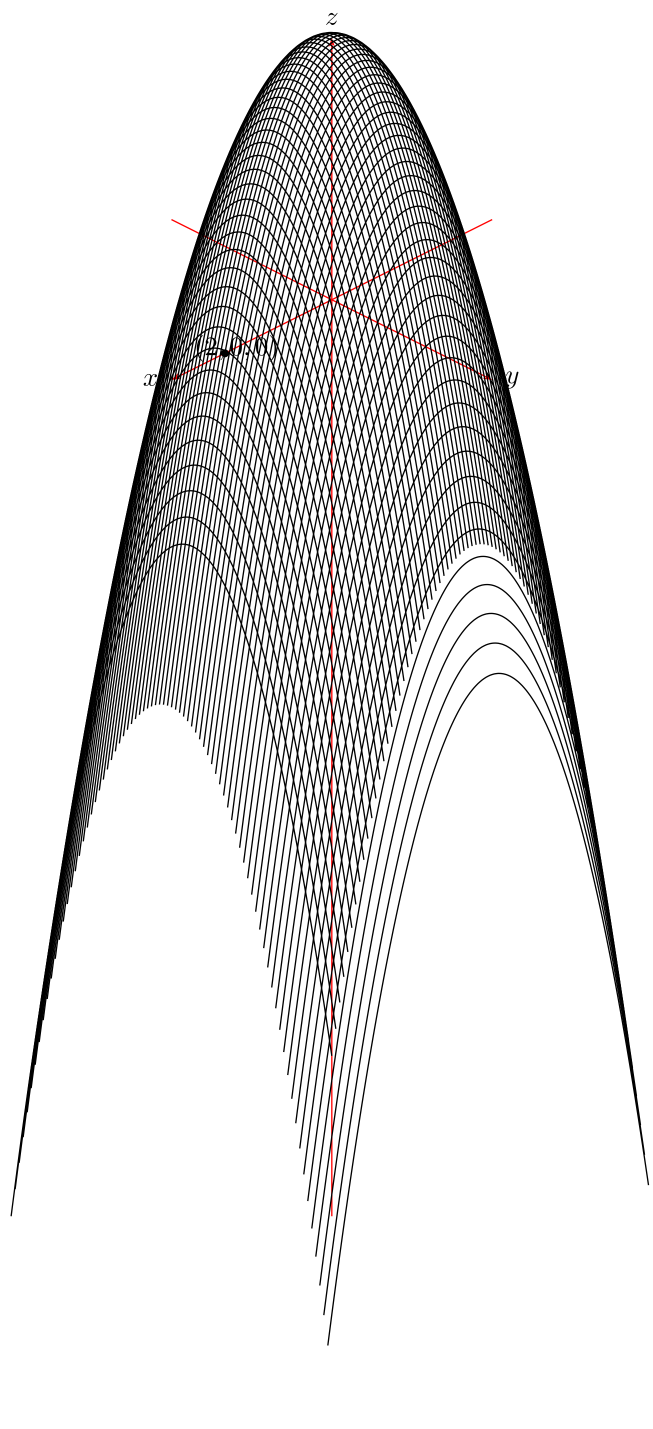
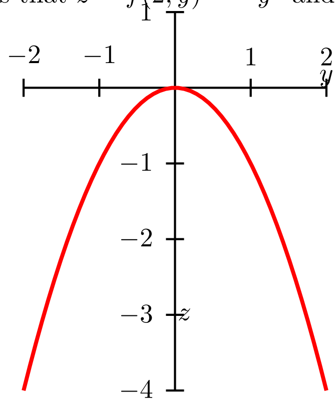
I thought that since sometimes the blackboard does not offer the fun amenities of the computer that I would use LaTeX to draw in something that would light the flames of knowledge in your mind.

With that let us again go back to our beloved example $f(x, y) = 4 - x^2 - y^2$.



If we want to really see what is in the plane $x = 2$ when it intersects the surface, substitute $x = 2$ into $f(x, y)$ to get $f(2, y) = 4 - 4 - y^2 = -y^2$. This means that $z = f(2, y) = -y^2$ and the curve in the plane $x = 2$ is



Thus we see that at the top of the parabola is the point $(2, 0, 0)$ or $(0, 0)$ in this drawing and we have that indeed $\frac{\partial f}{\partial y}(2, 0) = \lim_{h \to 0} \frac{f(2,h) - f(2,0)}{h} = 0$ since it is the slope of this curve at $(y = 0, z = 0)$.

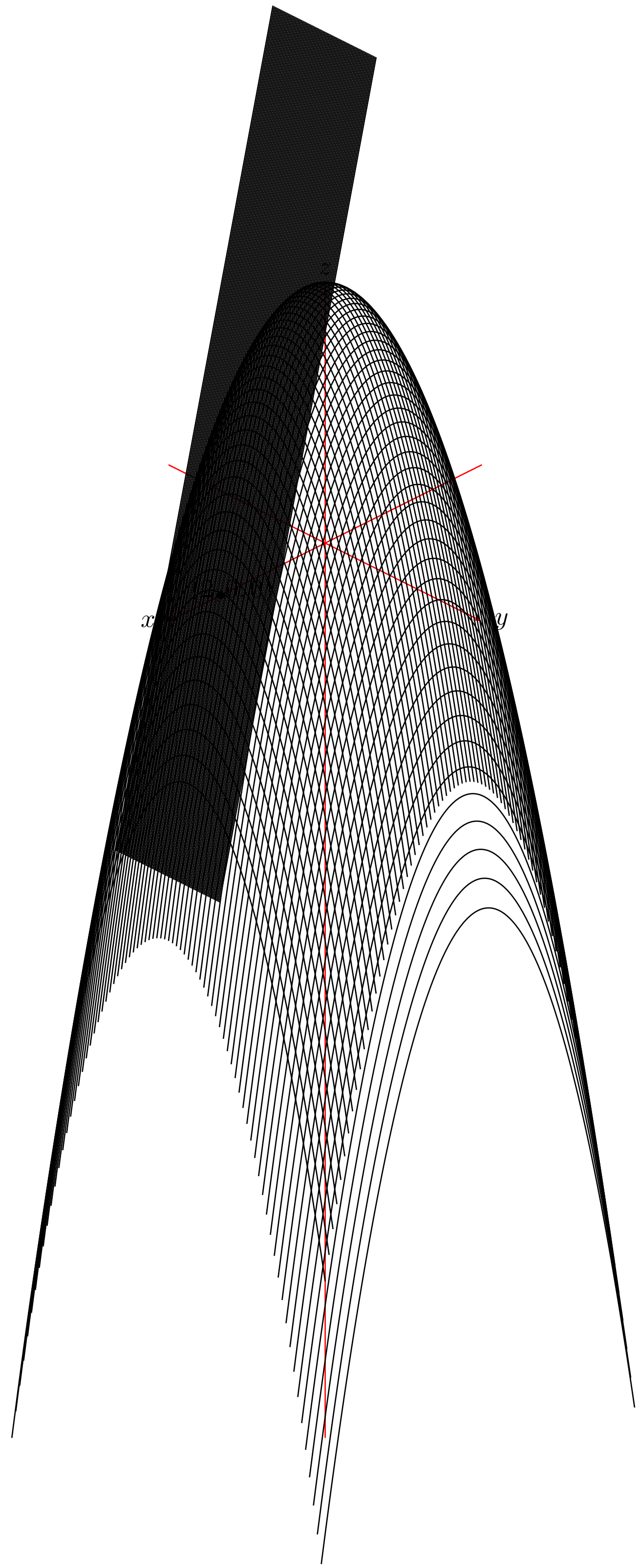
We can see that the tangent plane to $f(x, y)$ at $(2, 0)$ is

$$z - f(2, 0) = \frac{\partial f}{\partial x}(2, 0)(x - 2) + \frac{\partial f}{\partial y}(2, 0)(y - 0) \tag{1}$$

or

$$z = -4x + 8 \tag{2}$$

So we see the tangent plane at



Aren’t computers fun.