Homework 10.1-10.2

For exercise 3, 5 compute $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-2\mathbf{b}$, $3\mathbf{a}$ and $||5\mathbf{b}-2\mathbf{a}||$.

3.

$$\mathbf{a}=(2,4) \quad \mathbf{b}=(3,-1)$$

$$\mathbf{a}+\mathbf{b}=(2+3,4-1)=(5,3) \tag{1}$$

and

$$\mathbf{a}-2\mathbf{b}=(2,4)-2(3,-1)=(2-6,4+2)=(-4,6) \tag{2}$$

meanwhile

$$3\mathbf{a}=3(2,4)=(6,12) \tag{3}$$

and

$$5\mathbf{b}-2\mathbf{a}=5(3,-1)-2(2,4)=(15,-5)-(4,8)=(11,-13) \tag{4}$$

then we know that from (**??**)

$$||5\mathbf{b}-2\mathbf{a}||=\sqrt{11^2+(-13)^2}=\sqrt{121+169}=\sqrt{290} \tag{5}$$

5.

$$\mathbf{a}=\mathbf{i}+2\mathbf{j} \quad \mathbf{b}=3\mathbf{i}-\mathbf{j}$$

$$bva+\mathbf{b}=(1+3,2-1)=(4,1) \tag{6}$$

and

$$\mathbf{a}-2\mathbf{b}=(1,2)-2(3,-1)=(1-6,2+2)=(-5,4) \tag{7}$$

meanwhile

$$3\mathbf{a}=3(1,2)=(3,6) \tag{8}$$

and

$$5\mathbf{b} - 2\mathbf{a} = 5(3,-1) - 2(1,2) = (15,-5) - (2,4) = (13,-9) \tag{9}$$

then we know that from (**??**)

$$||5\mathbf{b} - 2\mathbf{a}|| = \sqrt{13^2 + (9)^2} = \sqrt{169 + 81} = \sqrt{250} = 5\sqrt{10} \tag{10}$$

In exercises 9 and 11 determine whether vectors **a** and **b** are parallel.

9.

$$\mathbf{a} = (2,1) \quad \mathbf{b} = (-4,-2)$$

$$\mathbf{a} = \frac{-1}{2}\mathbf{b} \tag{11}$$

11.

$$\mathbf{a} = (-2,3) \quad \mathbf{b} = (4,6)$$

If $\mathbf{a} = \frac{-1}{2}\mathbf{b}$ then $\mathbf{a}$ would equal $(-2,-3)$. On the other hand if $\mathbf{a} = \frac{1}{2}\mathbf{b}$ then $\mathbf{a}$ would equal $(2,3)$. Thus there is no way for $\mathbf{a} = c\mathbf{b}$ for any c an element of $\mathbb{R}$.

In exercises 15 and 17 find the vector $\overrightarrow{AB}$ for points A and B.

15.

$$A = (2,3) \quad B = (5,4)$$

$$\overrightarrow{AB} = (5-2, 4-3) = (3,1) \tag{12}$$

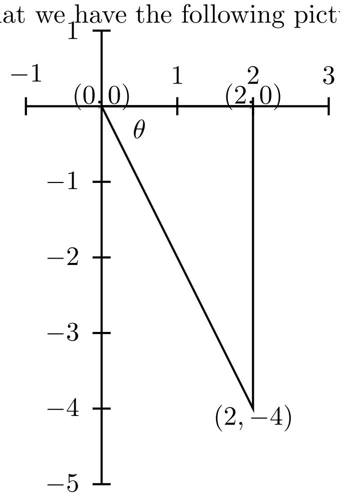
17.

$$A = (-1,2) \quad B = (1,-1)$$

$$\overrightarrow{AB} = (1-(-1), -1-2) = (2,-3) \tag{13}$$

In exercises 21 and 23 find a) a unit vector in the same direction as the given vector b) write the given vector in polar form.

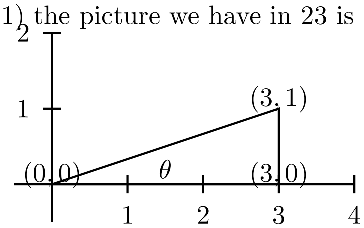
21. We shall have a lot of fun with polar form $\mathbf{v} = (2, -4)$ then we know that we have the following picture



This means that the magnitude of our vector is $2\sqrt{5}$ and that $\theta = \arctan(\tan(\theta)) = \arctan(-2)$. This, in turn, means that $\mathbf{v} = 2\sqrt{5}(\cos(\arctan(-2)), \sin(\arctan(-2))) = 2\sqrt{5}(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}})$.

Similarly a unit vector in the direction of $\mathbf{v}$ is $\mathbf{u} = (\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}})$.

23. After we use the fact that $A = (2, 1)$ and $B = (5, 2)$ to get that $\overrightarrow{AB} = (3, 1)$ the picture we have in 23 is



So we then have that the magnitude of our vector is $\sqrt{3^2 + 1} = \sqrt{10}$ and the angle $\theta = \arctan(\tan(\theta)) = \arctan(3)$. Then the unit vector in the direction of $\overrightarrow{AB} = (\cos(\arctan(3)), \sin(\arctan(3))) = (\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})$ and $\overrightarrow{AB} = \sqrt{10}(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})$.

In exercises 25 and 27 we shall find a vector with the given magnitude in the same direction as the given vector.

25. The magnitude is 3 and the vector $\mathbf{v} = (3, 4)$.

$$||\mathbf{v}|| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \tag{14}$$

Thus a new vector with magnitude 3 in the direction of $\mathbf{v}$ is $\frac{3}{5}\mathbf{v} = (\frac{9}{5}, \frac{12}{5})$

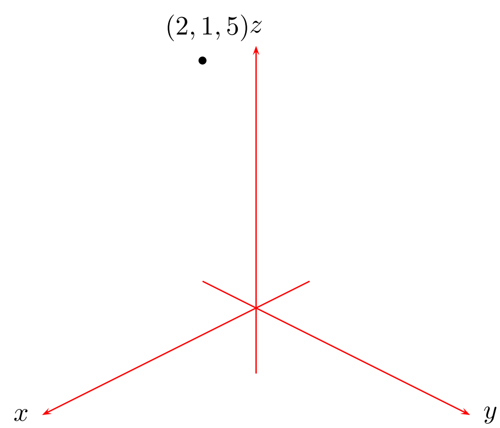
27. The magnitude is 29 and the vector is $\mathbf{v} = (2, 5)$.

$$||\mathbf{v}|| = \sqrt{2^2 + 5^2} = \sqrt{29} \tag{15}$$

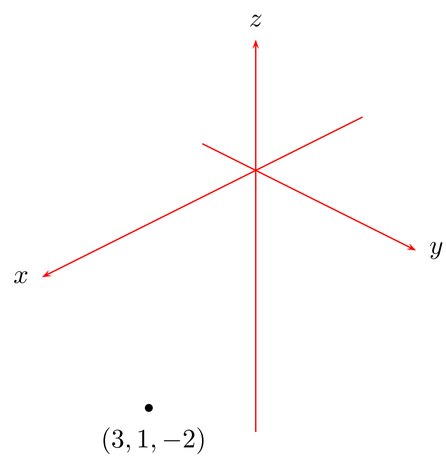
Hence taking $\frac{29}{\sqrt{29}}\mathbf{v} = \sqrt{29}\mathbf{v}$ we get a vector of magnitude 29 in the direction of $\mathbf{v}$ is $(2\sqrt{29}, 5\sqrt{29})$.

1. Plot the indicated points

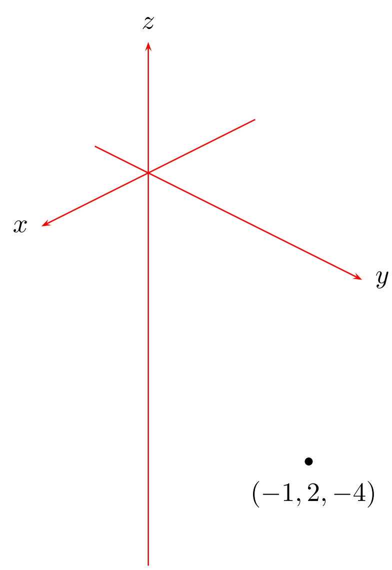
a. $(2, 1, 5)$


$(2, 1, 5)$
z
x
y

b. $(3, 1, -2)$



c. $(-1, 2, -4)$



In problem 5 find the distance between the given points

5. Point $A = (2, 1, 2)$ and $B = (5, 5, 2)$. We know that the vector $\overrightarrow{AB} = (3, 4, 0)$ and that it has length $\sqrt{3^2 + 4^2 + 0^2} = \sqrt{25} = 5$.

In exercise 11 compute $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - 3\mathbf{b}$, and $||4\mathbf{a} + 2\mathbf{b}||$.

11.

$$\mathbf{a} = (3, -1, 4) \quad \mathbf{b} = (5, 1, 0)$$

$$\mathbf{a} + \mathbf{b} = (3 + 5, 1 - 1, 4 + 0) = (8, 0, 4) = 4(2, 0, 1) \tag{16}$$

and

$$\mathbf{a} - 3\mathbf{b} = (3, -1, 4) - 3(5, 1, 0) = (3, -1, 4) - (15, 3, 0) = \tag{17}$$

$$(3 - 15, -1 - 3, 4 - 0) = (-12, -4, 4) = 4(-3, -1, 1) \tag{18}$$

and

$$4\mathbf{a}+2\mathbf{b} = 4(3, -1, 4)+2(5, 1, 0) = (12, -4, 16)+(10, 2, 0) = (22, -2, 16) = 2(11, -1, 8) \tag{19}$$

from (**??**) we have that

$$||4\mathbf{a} + 2\mathbf{b}|| = 2\sqrt{11^2 + 1 + 8^2} = 2\sqrt{186} \tag{20}$$

In exercise 13 a) find two unit vectors parallel to the given vector and b) write the given vector as the product of its magnitude and a unit vector.

13.

$$\mathbf{v} = (3, 1, 2)$$

First we find the magnitude of $\mathbf{v}$.

$$||\mathbf{v}|| = \sqrt{3^2 + 1 + 2^2} = \sqrt{14} \tag{21}$$

a). The two unit vectors are

$$\mathbf{u}_m \pm \frac{1}{\sqrt{14}}\mathbf{v} \tag{22}$$

for $m = \{1, 2\}$.

b) $\mathbf{v} = \sqrt{14}\mathbf{u}_1$.

In exercise 19 find a vector with the given magnitude and in the same direction as the given vector.

19. The magnitude is 6 and the vector is $\mathbf{v} = (2, 2, -1)$.

$$||\mathbf{v}|| = \sqrt{4+4+1} = \sqrt{9} = 3 \tag{23}$$

To have magnitude 6 we need that the vector have $\frac{6}{||\mathbf{v}||}\mathbf{v}$ as its coordinates. So the vector is

$$\frac{6}{||\mathbf{v}||}\mathbf{v} = 2(2, 2, -1) = (4, 4, -2) \tag{24}$$

In the exercises 23 find an equation of the sphere with radius r and center (a, b, c).

23.

$$r = 2 \quad (a, b, c) = (3, 1, 4)$$

$$(x-3)^2 + (y-1)^2 + (z-4)^2 = 4 \tag{25}$$

is the equation.

In exercise 27-30 identify the geometric shape described by the given equations.

27.

$$(x-1)^2 + y^2 + (z+2)^2 = 4$$

It is a sphere with radius 2 and center $(1, 0, -2)$.

29.

$$x^2 - 2x + y^2 + z^2 - 4z = 0 \tag{26}$$

$$x^2 - 2x + 1 + y^2 + z^2 - 4z + 4 = 5 \tag{27}$$

$$(x-1)^2 + y^2 + (z-2)^2 \tag{28}$$

It is a sphere with radius $\sqrt{5}$ and center $(1, 0, 2)$.

In exercises 31 identify the plane as parallel to the xy plane, xz plane, or yz plane and sketch a graph.

31. $y = 4$ is parallel to the xz plane

In exercise 35 give an equation for the indicated figure

35. xz-plane corresponds to $y = 0$.

43. Find the displacement vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ and determine whether the points $P = (2, 3, 1)$, $Q = (4, 2, 2)$, and $R = (8, 0, 4)$ are colinear.

$$\overrightarrow{PQ} = (4-2, 2-3, 2-1) = (2, -1, 1) \tag{29}$$

while

$$\overrightarrow{QR} = (8-4, 0-2, 4-2) = (4, -2, 2) = 2(2, -1, 1) \tag{30}$$

thus the since $\overrightarrow{PQ} = 2\overrightarrow{QR}$ we have that P, Q, and R are colinear.

45. Use vectors to determine whether the points $A = (0, 1, 1)$, $B = (2, 4, 2)$ and $C = (3, 1, 4)$

We know that

$$\overrightarrow{AB} = (2, 4-1, 2-1) = (2, 3, 1) \tag{31}$$

and that from this

$$||\overrightarrow{AB}|| = \sqrt{2^2 + 3^2 + 1} = \sqrt{4 + 9 + 1} = \sqrt{14} \tag{32}$$

Also

$$\overrightarrow{AC} = (3, 0, 3) \tag{33}$$

and that

$$||\overrightarrow{AC}|| = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \tag{34}$$

It cannot be an equilateral triangle because the lengths of two sides are not equal.

47. Use vectors and the Pythagorean theorem to determine whether the points $A = (3, 1, -2)$, $B = (1, 0, 1)$ and $C = (4, 2, -1)$ form a right triangle.

$$\overrightarrow{AB} = (1 - 3, -1, 1 - (-2)) = (-2, -1, 3) \tag{35}$$

where $||\overrightarrow{AB}|| = \sqrt{14}$.

and

$$\overrightarrow{AC} = (4 - 3, 2 - 1, -1 - (-2)) = (1, 1, 1) \tag{36}$$

where $||\overrightarrow{AC}|| = \sqrt{3}$.

finally

$$\overrightarrow{BC} = (4 - 1, 2 - 0, -1 - 1) = (3, 2, -2) \tag{37}$$

where $||\overrightarrow{BC}|| = \sqrt{17}$.

This means that since $||\overrightarrow{AC}||^2 + ||\overrightarrow{AB}||^2 = ||\overrightarrow{BC}||^2$ that by the law of cosines we have a right triangle in the plane spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

49. Use vectors to determine whether the points $A = (2, 1, 0)$, $B = (5, -1, 2)$, $C = (0, 3, 3)$ and $D = (3, 1, 5)$ form a square.

$$\overrightarrow{AB} = (5-2, -1-1, 2) = (3, -2, 2) \tag{38}$$

where $||\overrightarrow{AB}|| = \sqrt{17}$.

and

$$\overrightarrow{AC} = (0-2, 3-1, 3-0) = (-2, 2, 3) \tag{39}$$

where $||\overrightarrow{AC}|| = \sqrt{17}$.

finally

$$\overrightarrow{BC} = (0-5, 3-(-1), 3-2) = (-5, 4, 1) \tag{40}$$

where $||\overrightarrow{BC}|| = \sqrt{42}$.

However if $ABDC$ were a square we would have that $||\overrightarrow{BC}|| = \sqrt{34}$. Hence it is not a square.