

1. Find which line out of $l(t) = \langle 1 + 2t, -2 + 2t, 2 \rangle$ and $q(t) = \langle 1 + 2t, -2 + 3t, 2 - t \rangle$ is closer to the point $Q = (2, 0, 1)$.

On $l(t)$ we know that $l(0) = q(0) = P = (1, -2, 2)$. Also after picking a point $l(1) = R = (3, 0, 2)$ and $q(1) = S = (3, 1, 1)$ we can begin to find which has smallest distance to Q .

Look at the vectors $\overrightarrow{PQ} = \langle 1, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 2, 2, 0 \rangle$. We begin by computing their cross product.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 2 & 2 & 0 \end{vmatrix} = \mathbf{k} \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} \\ 2 & 2 \end{vmatrix} = 2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k} \quad (1)$$

Then from (1) we have

$$\frac{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|}{\|\overrightarrow{PR}\|} = \frac{2\sqrt{3}}{2\sqrt{2}} = \sqrt{\frac{3}{2}} = d(Q, l(t)) \quad (2)$$

Now taking $\overrightarrow{PS} = \langle 2, 3, -1 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 2 & 3 & -1 \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 1 & 1 & 0 \end{vmatrix} \quad (3)$$

Since adding a multiple of one row of a matrix to another does not change the determinant.

This means that (3) equals

$$\mathbf{k} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} \\ 1 & 1 \end{vmatrix} = \mathbf{i} - \mathbf{j} - \mathbf{k} \quad (4)$$

From (4) we know that

$$\frac{\|\vec{PQ} \times \vec{PS}\|}{\|\vec{PS}\|} = \frac{\sqrt{3}}{\sqrt{14}} = \sqrt{\frac{3}{14}} = d(Q, q(t)) \quad (5)$$

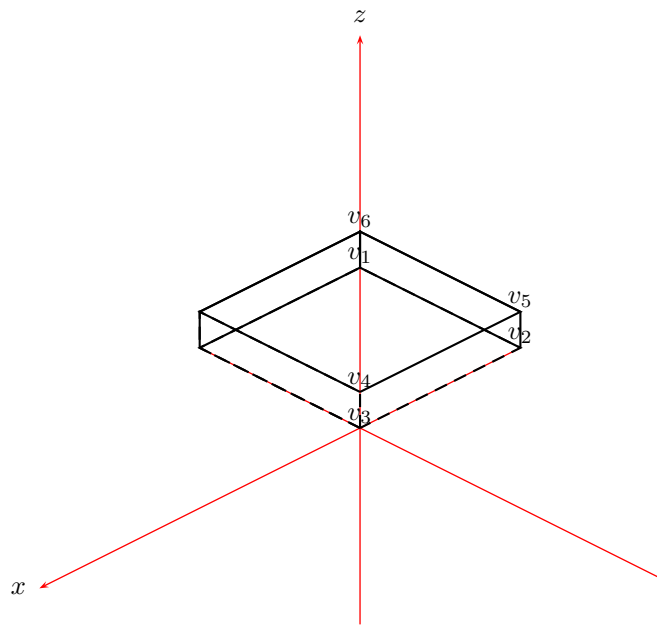
Hence since $d(Q, q(t)) < d(Q, l(t))$ we know that Q is closer to $q(t)$.

52. Find the volume of the parallelepiped with 3 adjacent edges formed by $\mathbf{v} = \langle 0, -1, 0 \rangle$, $\mathbf{w} = \langle 0, 2, -1 \rangle$ and $\mathbf{t} = \langle 1, 0, 2 \rangle$.

Take

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix} = \mathbf{i} \quad (6)$$

Then taking $|\mathbf{t} \cdot \mathbf{i}| = 1$ is the volume of the parallelepiped



Find the volume enclosed in the vertices $v_1 = (-1, -1, 0)$, $v_2 = (-1, 0, 0)$, $v_3 = (0, 0, 0)$, $v_4 = (1, 1, 1)$, $v_5 = (0, 1, 1)$ and $v_6 = (0, 0, 1)$.

Since this is a triangular half of a parallelogram we will find the volume of the parallelogram enclosed by $\vec{p} = \langle -1, 0, 0 \rangle$, $\vec{q} = \langle 0, -1, 0 \rangle$ and $\vec{r} = \langle 1, 1, 1 \rangle$.

Then taking

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \mathbf{k} \quad (7)$$

Hence we have that $|\vec{r} \cdot \mathbf{k}| = 1$. This means that the volume of the triangular solid we are interested in is $\frac{1}{2}$.

63. Show that $\|\mathbf{a} \times \mathbf{b}\|^2 = \|\mathbf{a}\|^2\|\mathbf{b}\|^2 - (\mathbf{a} \cdot \mathbf{b})^2$. Use this to show that $\|\mathbf{a} \times \mathbf{b}\|^2 + \mathbf{a} \cdot \mathbf{b} = (\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b})$.

We know that $\|\mathbf{a} \times \mathbf{b}\|^2 = \|\mathbf{a}\|^2\|\mathbf{b}\|^2 \sin^2(\theta)$ and similarly $(\mathbf{a} \cdot \mathbf{b})^2 = \|\mathbf{a}\|^2\|\mathbf{b}\|^2 \cos^2(\theta)$ use this and $\cos^2(\theta) + \sin^2(\theta) = 1$ and you are done.