

$K(2)$ -local homotopy theory

Research Among Peers (RAP), Fall 2004

One of the basic tools of homotopy theory is that of *localization*. If E is a generalized homology theory, the E -local homotopy category of spaces (or of spectra) is the category obtained from the homotopy category of spaces by formally inverting the E -equivalences, i.e., inverting maps which induce an isomorphism on the E -homology groups. This leads to a strategy for understanding the homotopy category by fracturing it into simpler problems: (1) understand the E -local homotopy category for various E , and (2) patch the E -local information of part (1) together to understand the original homotopy category. For the stable homotopy category, this is called the *chromatic* viewpoint; the basic building blocks are the $K(n)$ -local categories, where $K(n)$ denotes the n th Morava K -theory.

The goal of this seminar is to understand how to do calculations in the $K(2)$ -local homotopy category. The eventual aim is to be able to understand something of the papers of Goerss, Henn, Mahowald, et al, [GHM], [GHMR], and the paper of Behrens [B], which all purport to give methods which should allow the calculation of the $K(2)$ -local sphere.

We will start with a crash course on localization theory and the chromatic viewpoint, using the papers of Bousfield [Bou79] and Ravenel [Rav84] and the book of Ravenel [Rav92] as jumping off points, with special attention as to the way $K(n)$ -local homotopy theory fits into the picture. We will then look at $K(1)$ -local homotopy theory, which is closely related to K -theory and is well understood. We will then set up the machinery needed to study general $K(n)$ -local homotopy theory, before proceeding to the papers on the $K(2)$ -local category mentioned above. We envisage that it will take two semesters to get to where we wish to go.

We will have an organizational meeting on Tuesday, August 31, at time and place to be announced. Tentatively, we plan to meet one a week for lectures and once for an informal discussion section. Lecturing duties will be divided amongst the participants. Course credit for active participation may be available if desired (pending chairman's approval of this RAP).

Please email Charles Rezk (rezk@math.uiuc.edu) if you are interested in participating.

References

[GHM] P. Goerss, H.-W. Henn, M. Mahowald, "The homotopy of $L_2V(1)$ at the prime 3", online.

[GHMR] P. Goerss, H.-W. Henn, M. Mahowald, C. Rezk, "A resolution of the

$K(2)$ -local sphere ”, online.

[B] M. Behrens, “A modular description of the $K(2)$ -local sphere”, online

[Bou79] A. K. Bousfield, “The localization of spectra with respect to homology”, *Topology* **18** (1979), 257–281.

[Rav84] D. C. Ravenel, “Localization with respect to certain periodic homology theories”, *Amer. J. Math* **106** (1) (1984), 351–414.

[Rav92] D. C. Ravenel, *Nilpotence and periodicity in stable homotopy theory*, *Annals of Mathematics Studies*, Princeton University Press, 1992.