A Few Review Problems for Exam 2

Math 225

November 4, 2008

1. Let A and B be $n \times n$ matrices with det $A = 2$, det $B = 3$. Find, if possible, det $2A$, det $(-B)$, det $(A+B)$, det (ABA^2B^2), det (adj A), det (adj B).
   det $2A = 2^n$det $A = 2^{n+1}$
   det $-B = 3(-1)^n$
   det $(A+B)$=?
   det (ABA^2B^2)= $2^3 3^3$
   det (adj A)= 2^{n-1}
   det (adj B)= 3^{n-1}

2. Is $S = \{(x_1,\ x_2,\ x_3,\ x_4)^T :\ x_1 + x_2 = x_3 + x_4\}$ a subspace of $\mathbb{R}^4$? If so, then find a basis and dim S.
   Yes, dim $S = 3$. $\mathcal{B} = \{[-1\ 1\ 0\ 0]^T,\ [1\ 0\ 1\ 0]^T,\ [1\ 0\ 0\ 1]^T\}$

3. Is $S = \{(x_1,\ x_2,\ x_3,\ x_4)^T :\ x_1 + x_2 + 3 = x_3 + x_4\}$ a subspace of $\mathbb{R}^4$? If so, then find a basis and dim S.
   No, since the $\mathbf{0}$ vector is not present.

4. Is $S = \{p(t) = a + bt + ct^2 + dt^3 + et^4 :\ p(0) = 0\}$ a subspace of $\mathbb{P}_4$? If so, then find a basis and dim S.
   Yes, dim $S = 4$. Basis is $\mathcal{B} = \{t,\ t^2,\ t^3,\ t^4\}$.

5. Is $S = \{p(t) = a + bt + ct^2 + dt^3 + et^4 :\ p(1) = p(2)\}$ a subspace of $\mathbb{P}_4$? If so, then find a basis and dim S.
   Yes, dim $S = 4$. Basis is $\mathcal{B} = \{1,\ t^2 - 3t,\ t^3 - 7t,\ t^4 - 15t\}$.

6. Is $S = \{p(t) = a + bt + ct^2 + dt^3 + et^4 :\ all\ coefficients\ are\ integers\}$ a subspace of $\mathbb{P}_4$? If so, then find a basis and dim S.
   No, since scalar multiples are not always present.

7. Find Rank A, and basis and dimensions of Col A, Row A, and Nul A, if

$$A_1 = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 3 & 3 & 3 \end{bmatrix} \qquad or \qquad A_2 = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 0 & 2 \\ 4 & 3 & 1 & 3 \end{bmatrix}$$

In both cases Rank A = dim Col A = dim Row A = dim Nul $A = 2$.

8. Consider the vectors below:

$$\begin{vmatrix} 1 \\ 0 \\ 1 \\ 2 \end{vmatrix}, \quad \begin{vmatrix} 2 \\ 1 \\ 1 \\ 1 \end{vmatrix}, \quad \begin{vmatrix} 1 \\ 0 \\ 1 \\ 0 \end{vmatrix}, \quad \begin{vmatrix} 4 \\ 1 \\ 3 \\ 5 \end{vmatrix}$$

   (i) Extend (if possible) $\{v_1,\ v_2,\ v_3\}$ to a basis of $\mathbb{R}^4$.
   Put vectors $v_1,\ v_2,\ v_3$ into a matrix with the vector $[a\ b\ c\ d]^T$. Reduce to find that the matrix is invertible when $c-a+b \neq 0$. Choosing any such vector will make a basis by IMT. For example, I chose $[1\ 2\ 4\ 0]^T$.

   (ii) Are $v_1,\ v_2,\ v_4$ linearly independent?
   No, there is a free variable when we reduce.

   (iii) Is Span$\{v_1,\ v_2,\ v_3,\ v_4\} = \mathbb{R}^4$?
   No, since the vectors are linearly dependent (by IMT). We need 4 linearly independent vectors to Span $\mathbb{R}^4$.

9. If the null space of a 7×5 matrix A is 4-dimensional, what is the dimension of the column space of A?
   dim Nul A + dim Col $A = 5$, so dim Col $A = 1$.

10. Compute the adjugate of the given matrix, if possible:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 3 & 2 \end{bmatrix}$$

$$adj A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 6 & 0 \\ -1 & -9 & 3 \end{bmatrix}$$

11. Let A and P be square $n \times n$ matrices, with P invertible. Show that det (PAP^{-1}) =det A.
    det (PAP^{-1}) =(det P)(det A)(det (P^{-1})) =(det P)(det A)$\frac{1}{(det\ P)}$ =det A.
    The first step is true by Theorem 6 in 3.2. The second step is true by HW 31 in 3.2. Then the last step is true by properties of multiplication.

12. Suppose that A is a square matrix such that det $A^4 = 0$. Explain why A cannot be invertible.
    By Theorem 6 in 3.2, det $A^4 = 0$ means (det $A)^4 = 0$, so det $A = 0$ and thus A is not invertible.

13. Compute the determinants of the following matrices:
det $A = -22$, det $B = 24$, and det $C = -30$.

14. Use Cramer's Rule to solve the following system of equations:
det $A = 1$, det $A_1(b) = 1$, det $A_2(b) = -3$, so solution is (1, -3).