

Review Problems for the Fifth Midterm Math 220 U1 — December 3, 2007

Work the following problems in your group and on your own. Check the class website for solutions.

1. Find the area bounded by the graphs of $y = x^2$ and $y = 8 - x^2$ for $0 \leq x \leq 4$.
2. Find the area bounded by $y = x^2$, $y = 6 - x$, and $y = 0$.
3. Find the volume of an object whose base is the region in the xy -plane bounded by $y = x^2$ and $y = 4$, with its cross-sections parallel to the x -axis all in the form of squares.
4. Find the volume of the solid of revolution formed by revolving around the line $y = -1$ the region bounded by $y = 4 - x^2$ and the x -axis.
5. Find the volume of the solid generated by revolving about the line $x = 7$ the region bounded by $y = x$ and $y = x^2 - 42$.
6. Find the volume of the solid generated by revolving around the y -axis the region bounded by $y = \ln x$, $y = 0$, and $y = 1$. (Hint: use the disc/washer method.)
7. Find the volume of the solid generated by revolving around the y -axis the region bounded by $y = \sin(x^2)$ and the x -axis for $0 \leq x \leq \pi$. (Hint: use the shell method.)
8. Write an integral representing the length of the curve $y = \ln x$ between $x = 1$ and $x = e$. Do not evaluate the integral.
9. Find the length of the curve $y = x^{3/2}$, for $0 \leq x \leq 1$. Completely evaluate any integral you obtain.
10. Find the area of the surface swept out by revolving the line segment $y = x + 1$, $0 \leq x \leq 1$, about the x -axis. Completely evaluate any integrals you obtain.
11. Write, but do not evaluate, an integral representing the surface area swept out by the revolving the parabola $y = x^2$, $0 \leq x \leq 2$, about the x -axis.
12. A cannonball is launched from an initial height of zero with initial speed 100 ft/sec at an angle of $\pi/6$. How far (horizontally) does the cannonball travel before it hits the ground (assuming the ground is level)? Assume that the gravitational acceleration is 32 ft/sec² downward.
13. An object is launched vertically from the ground. What should the object's initial speed be in order to make sure the object is in the air for exactly 3 seconds?
14. A rod extending from $x = 0$ to $x = 3$ has density given by $\rho(x) = \sqrt{x + 1}$. Find the center of gravity of the rod.)
15. A rod extending from $x = 0$ to $x = 6$ has density given by $\rho(x) = 5x$. Find the center of gravity of the rod.