Name: ______________________________________

Math 231 W3, Spring Semester 2008

Mock Exam # 3

April 16, 2008

Problem 1: Determine the interval of convergence for the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^k k}(2x-1)^k.$$

Problem 2: Suppose

$$f(x) = \sqrt{x}.$$

(a) Find the fourth-degree Taylor polynomial $P_3(x)$ for $f(x)$ expanded about $c = 4$.

(b) Find the remainder term $R_3(x)$ that goes with the Taylor polynomial in part (a).

Problem 3: Find the first five nonzero terms of the Maclaurin series for both of the following functions.

(a) $\sqrt{1-2x}$

(b) $\int \sin(x^2)\, dx$

Problem 4: (a) Find the point(s) (x, y) where the two parametric curves

$$x = t + 3, \quad y = t^2, \quad -\infty < t < \infty \qquad \textbf{and} \qquad x = 1 + s, \quad y = 2 - s, \quad -\infty < s < \infty$$

intersect, if they ever do.

(b) Assuming that s and t measure time from the same instant and on the same scale (i.e., $s = t$ always), do two objects following these curves ever *collide*? If so, then where, and at what time?

Problem 5: The parametrized curve

$$x = 3\cos t, \qquad y = 2\sin t$$

traces out an ellipse.

(a) Find the area of the ellipse.

(b) Find the second derivative dy^2/dx^2 of the curve at the point $(0, -2)$.

Problem 6: Suppose we're given the curve

$$x = t, \qquad y = t^3, \qquad -\infty < t < \infty.$$

(a) Set up, but do not evaluate, an integral representing the length of the curve between the points $(0,0)$ and $(2,8)$.

(b) Set up <u>and evaluate</u> and integral representing the area of the surface generated when we revolve the portion of the curve between $x = 0$ and $x = 1$ around the x-axis.

Review Problems for the Final — Sections 7.1 and 8.1

These problems are provided in preparation for your final. They are typical of what you can expect from Sections 7.1 and 8.1. As before, in an effort to motivate you to work with your classmates and not postpone thinking about these problems, I will *not* be posting solutions to these problems. If you get stuck in working a problem, let me or a fellow class member help you out. Good luck!

Find the solution of the given differential equation satisfying the indicated initial condition.

A. $y' = 4y,\ y(0) = 2$ B. $y' = 3y,\ y(0) = -2$ C. $y' = 2y,\ y(1) = 2$

D. Suppose a bacterial culture triples in population every 5 hours. If the population is initially 200, find an equation for the population at time t. Determine when the population will reach 20,000.

E. A scientist observes a radioactive substance. Initially, there are 300 grams present. After 4 hours, the substance has decayed until there are only 100 grams present. Find the half-life of the substance.

Find the limit of each sequence, if it exists.

F. $a_n = \dfrac{1}{n^3}$ G. $a_n = \dfrac{4}{n!}$ H. $a_n = \dfrac{3n^2+1}{2n^2-1}$

I. $a_n = (-1)^n \dfrac{n+4}{n+1}$ J. $a_n = ne^{-n}$ K. $a_n = \dfrac{\cos n}{n^2}$