Merit Worksheet #2, 1/16/08

Integration by Parts

1. Look at the integral
$$\int x \cos x \, dx.$$
Say you decide to try to find this integral through integration by parts.
   (a) Write down at least three possible choices for u and dv.
   (b) For each choice of u and dv you came up with in part (a), what does the integration by parts formula give you?
   (c) Choose the best-looking answer to part (b), and find the answer to the integral.

2. Find the following integrals using integration by parts:
$$\text{(a)} \quad \int xe^{2x} \, dx \qquad\qquad \text{(b)} \quad \int \arctan 3x \, dx$$

3. Why is it never a good idea, when applying integration by parts to $\int f(x) \, dx$, to let $u = 1$ and $dv = f(x) \, dx$?

4. Find the following integrals by integration by parts:
   (a) $\int x \sin 2x \, dx$
   (b) $\int_1^e x^2 \ln x \, dx$
   (c) $\int_0^2 t^2 e^{5t} \, dt$
   (d) $\int_0^1 x \arcsin(x^2) \, dx$

5. Find the integral
$$\int e^{3x} \cos 2x \, dx.$$
(Hint: you'll find it helpful to integrate by parts *twice*. Pay close attention to what you arrive at the second time around.)

6. Look at the second integration by parts you did in Problem 5. What would happen if you switched u and dv? Why does this happen? Is there a moral to this story?

7. Opening up a calculus book at random, you see the *reduction formula*
$$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx.$$
   (a) Use the formula to find $\int x^3 e^x \, dx$.
   (b) How did the book derive this formula?

8. Look at the integral
$$\int_{-1}^{2} y\sqrt{y+2} \, dy.$$
   a) Find this integral by integration by parts.
   b) Find this integral through a u-substitution.

c) Are your answers to (a) and (b) equal?

9. Find the integral

$$\int e^{-\sqrt{x}}\,dx.$$

(Hint: make a substitution first, and use integration by parts on the result.)

Reading assignment for Friday, 1/18: Next time we'll cover the material on pages 521 to 525 of your text (note that this is only half of Section 6.3). Read with attention all the text of the section through Example 6.1. Make sense of Examples 3.4 and 3.6 (and read the paragraphs just before each one). Skim the rest of the material. Prepare the solution to Exercise 1 to turn in; make sure to show *all* your steps.

Proofs to impress your teachers and amaze your friends: Let's apply integration by parts to the integral

$$\int \frac{1}{x}\,dx,$$ [1]

with $u = 1/x$ and $dv = 1\,dx$. We get

$$\int \frac{1}{x}\,dx = \frac{1}{x}\cdot x - \int x\cdot -\frac{1}{x^2}\,dx = 1 + \int \frac{1}{x}\,dx.$$

Subtracting $\int \frac{1}{x}\,dx$ from both sides, we find that

$$\boxed{0 = 1}.$$

Any questions?

[1] Yes, I know that we don't *need* to use integration by parts to get the answer. Just humor me for a bit.