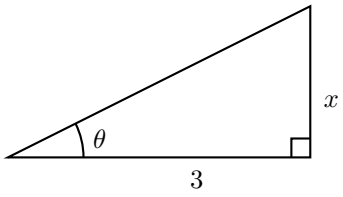
Selected answers — Merit Worksheet #4

1.



2. (a) $x = \sin\theta$, $dx = \cos\theta d\theta$

   (b) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

   (c) Making x into $\sin\theta$ creates the quantity $1 - \sin^2\theta$, which perfectly fits a trig identity.

3.

(a) $\displaystyle\ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C$ (b) $\displaystyle\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| - \frac{\sqrt{x^2-4}}{x} + C$ (c) $\displaystyle-\frac{\sqrt{1-4x^2}}{x} + C$

4. $\displaystyle\frac{1}{4}\pi R^2 + \frac{a}{2}\sqrt{R^2-a^2} + \frac{1}{2}R^2 \arctan\left(\frac{a}{\sqrt{R^2-a^2}}\right)$

5. πab.

6. (a) Let $u = 1 + x^2$; the integral becomes $\displaystyle\frac{1}{2}\int_1^2 (u-1)\sqrt{u}\,du$, and the answer is $\displaystyle\frac{2\sqrt{2}+2}{15}$

   (b) Let $x = \tan\theta$; the integral simplifies to $\displaystyle\int \tan^3\theta \sec^3\theta\,d\theta$, and the answer once again is $\displaystyle\frac{2\sqrt{2}+2}{15}$.

7.

(a) $\displaystyle\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$ (b) $\displaystyle\frac{x}{\sqrt{1-x^2}}$ (c) $\displaystyle\sqrt{x^2-1} - \sec^{-1} x + C$

8. (a) For $x^2 + a^2$, use $x = a\sinh t$. For $x^2 - a^2$, use $x = a\cosh t$.

   (b)

(a) $\displaystyle\sinh^{-1}\left(\frac{x}{3}\right) + C$ (b) $\displaystyle\cosh^{-1}\left(\frac{x}{2}\right) - \frac{\sqrt{x^2-4}}{x} + C$