

Merit Worksheet #5, 1/25/08

A bunch of integral problems

1. Evaluate each of the following antiderivatives *in at least two separate ways*:

$$(a) \int \frac{x^2}{\sqrt{1-x}} dx \quad (b) \int \frac{1}{x(\ln x)^3} dx \quad (\text{assume } x > 0) \quad (c) \int \frac{\ln x}{x} dx$$
$$(d) \int \sin x \cos x dx \quad (e) \int \frac{x}{9+x^2} dx$$

2. So you think the integral $\int \sec x dx$ comes out of nowhere? Let's take another look at it.

- (a) Write down (from memory) what the antiderivative is.
(b) Derive the antiderivative again, this time by (i) writing $\sec x$ as $1/\cos x$, (ii) multiplying both numerator and denominator by $\cos x$, (iii) making a u -substitution, and finally (iv) using the fact that

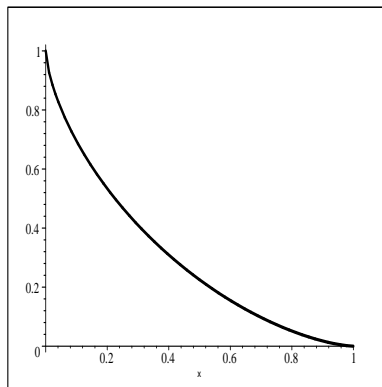
$$\frac{1}{1-u^2} = \frac{1}{2} \left(\frac{1}{1+u} + \frac{1}{1-u} \right).$$

- (c) (bonus) Derive the antiderivative again, this time by (i) writing $\sec x$ as $1/\cos x$, (ii) writing $\cos x$ as $\sin(x + \pi/2)$, (iii) making the substitution $u = x + \pi/2$, (iv) using the identity $\sin 2\theta = 2 \sin \theta \cos \theta$, (v) making the substitution $w = u/2$, (vi) multiplying the integrand by $(\sec^2 w)/(\sec^2 w)$, and (vii) making the substitution $U = \tan w$.

3. Evaluate the following integrals:

$$(a) \int (\ln x)^2 dx \quad (b) \int \tan x \sec^3 x dx \quad (c) \int \frac{dx}{(5-x^2)^{3/2}}$$
$$(d) \int \cos \sqrt{x} dx \quad (e) \int \ln(1+x^2) dx \quad (f) \int \frac{1}{e^x \sqrt{4+e^{2x}}} dx$$
$$(g) \int x \sqrt{6x-x^2-8} dx$$

4. Suppose you took a paintbrush with width 1 unit and propped it up along the y -axis, with its bottom edge at the origin. If you slide the bottom edge of the paintbrush along the x -axis and let the top edge slide down the y -axis, the area painted by the brush is bounded by the x -axis, the y -axis, and the curve $y = (1-x^{2/3})^{3/2}$, shown below:



- (a) Find the area painted by the brush.
- (b) (bonus) Show that the top edge of the painted area has equation $y = (1 - x^{2/3})^{3/2}$.

Reading assignment for Monday, 1/28: We'll move on next time to Section 6.4. Read with attention all the text through Example 4.2, and Remark 4.1 on page 532. Also read the "Brief Summary of Integration Techniques" portion at the end of the section. Skim the rest of the section, and write up Exercise 1 to turn in next time.

Joke of the day: According to Wikipedia, Norbert Wiener (1894-1964) "was an American theoretical and applied mathematician. He was a pioneer in the study of stochastic and noise processes, contributing work relevant to electronic engineering, electronic communication and control systems. Wiener is perhaps best known as the founder of cybernetics, a field that formalizes the notion of feedback and has implications for engineering, systems control, computer science, biology, philosophy, and the organization of society." That's a lot of accomplishments, but what some people remember most about Norbert Wiener was his absent-mindedness, which was legendary. Consider the following:

One day the Wiener family was scheduled to move into a new house. Mrs. Wiener, mindful of her husband's propensity for forgetting, wrote the new address on a slip of paper and handed it to him. He scoffed, saying, "I wouldn't forget such an important thing," but he took the slip of paper and put it in his pocket. Later that same day at the university a colleague came by his office with an interesting problem. Wiener searched for a piece of paper and took the slip from his pocket to use to write some mathematical equations. When he finished, he crumpled up the slip of paper and threw it away. That evening, he remembered there was something about a new house but he couldn't find the slip of paper with the address on it. Without any alternative course of action, he returned to his old home, where he spotted a little girl on the sidewalk. "Say, little girl," he said, "Do you know where the Wieners live?" The girl replied, "That's o.k., Daddy, Mommy sent me to get you."

(Retold in "Foolproof: A Sampling of Mathematical Folk Humor," *Notices of the American Mathematical Society*, 52 (2005), no. 1, pg. 32.)