Merit Worksheet #7, 1/30/08

More practice with partial fractions

1. Determine the partial fraction decomposition of each given rational function.

   (a) $r(x) = \dfrac{x^2 - 3x - 1}{x^3 + x^2 - 2x}$

   (b) $r(x) = \dfrac{2x^2 + 3}{x^2(x - 1)}$

   (c) $r(x) = \dfrac{x}{x^4 - 1}$

   (d) $r(x) = \dfrac{x^4}{(x - 1)^3}$

2. Evaluate the given integrals:

   (a) $\displaystyle\int \frac{1}{t([\ln t]^2 - 4)}\, dt$

   (b) $\displaystyle\int \frac{\sec^2\theta}{\tan^3\theta - \tan^2\theta}\, d\theta$

   (c) $\displaystyle\int \frac{dt}{e^t + 5 + 6e^{-t}}$

A brief stab at a real-world application

3. In honor of the Superbowl, suppose we create a football by revolving the curve $y = \sin x$ between $x = 0$ and $x = \pi$ about the x-axis.

   (a) Referring to Section 5.2 of your text as necessary, find the volume of such a football.

   (b) Referring to Section 5.4 of your text as necessary, find the surface area of the football.

A hodgepodge of integrals

Section 6.5 in your text, which we won't be covering, deals with how to use integral tables (which are fairly self-explanatory) and computer algebra systems (abbreviated CAS's, which your textbook authors love to dwell on) to solve integrals. You may find it interesting to skim this section. However, try *not* to use either a table or a CAS in the following problems.

4. Find the following integrals (the starred problems may or may not be a bit more of a challenge):

   (a) $\displaystyle\int_0^1 \frac{x^3}{1 + x^4}\, dx$ (b) $\displaystyle\int \sec^3 x\, dx$ (c) $\displaystyle\int \tan^{-1} x\, dx$ (d) $\displaystyle\int \frac{x^2 + 3}{x^2 - 3x + 2}\, dx$

   (e) $\displaystyle\int \sin(\ln x)\, dx$ (f*) $\displaystyle\int e^{2x}\sqrt{1 + e^x}\, dx$ (g*) $\displaystyle\int x \sin^2 x\, dx$ (h*) $\displaystyle\int_0^1 x^8\sqrt{4 - x^6}\, dx$

Reading assignment for Friday, 2/1: On Friday we will cover the first half or so of Section 6.6 on improper integrals. Read the section through the end of page 547, and the following items: Definition 6.1 and Examples 6.2, 6.3, and 6.4; and Definition 6.2 and Example 6.5. Read the section "Improper Integrals with an Infinite Limit of Integration" up through Example 6.7. Skim, at least, the rest of the section through Example 6.10, and prepare Exercise 1 to turn in on Friday.

Joke of the day: Monday evening you may have watched the annual State of the Union address. The following is a joke that's been circulating on the internet for years now—it's a little dated, and it may show some political bias (not necessarily my own), but it's good for a laugh.[1] Enjoy!

> At New York's Kennedy airport today, an individual (later discovered to be a public school teacher) was arrested trying to board a flight while in possession of a ruler, a protractor, a compass, and a calculator. At a morning press conference, Attorney general John Ashcroft said he believes the man is a member of the notorious al-Gebra movement. He is being charged by the FBI with carrying weapons of math instruction.
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> "Al-Gebra is a fearsome cult," Ashcroft said. "They desire average solutions by means and extremes, and sometimes go off on tangents in a search of absolute value. They use secret code names like 'x' and 'y' and refer to themselves as 'unknowns,' but we have determined they belong to a common denominator of the axis of medieval with coordinates in every country. As the Greek philanderer Isosceles used to say, there are 3 sides to every triangle," Ashcroft declared.
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> When asked to comment on the arrest, President Bush said, "I am gratified that our government has given us a sine that it is intent on protracting us from those who are willing to disintegrate us with calculus disregard. Murky statisticians love to inflict plane on every sphere of influence," the President said, adding: "Under the circumferences, we must differentiate their root, make our point, and draw the line."
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> President Bush warned, "These weapons of math instruction have the potential to decimal everything in their math on a scalene never before seen unless we become exponents of a higher power and begin to factor-in random facts of vertex."
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> Attorney General Ashcroft said, "As our great leader would say, read my ellipse. Here is one principle he is uncertainty of: though they continue to multiply, their days are numbered as the hypotenuse tightens around their necks."

[1] In fact, this is the best math joke I have. It's all downhill from here, I'm sorry to say.