Merit Worksheet #10, 2/6/08

Review of improper integrals and the comparison test

1. The results we covered at the beginning of Monday's worksheet are sometimes called *the p-test*. Let's review them again—fill in the blanks:

   The integral $\int_0^1 \frac{1}{x^p}\,dx$ converges when...

   The integral $\int_1^\infty \frac{1}{x^p}\,dx$ converges when...

2. Find all values of p for which the integral

$$\int_0^\infty \frac{1}{x^p}\,dx$$

converges.

Differential equations and exponential growth and decay

3. Solve the following differential equations:
   (a) $y' = 3y$, $y(0) = -2$
   (b) $y' = -2y$, $y(0) = -6$
   (c) $y' = -y$, $y(1) = 2$

4. Suppose $\$P$ is deposited in a bank account paying $r\%$ interest compounded continuously.
   (a) Write a differential equation modeling this situation.
   (b) What is the solution to the differential equation you wrote in part (a)?
   (c) If \$4500 is deposited in a bank account paying 8% compounded continuously, then what amount will be in the account at the end of 6 years?

5. Suppose a bacterial culture triples in population every 5 hours. If the population is initially 200, find an equation for the population at any time. Determine when the population will reach $20,000$.

6. Suppose some quantity is increasing exponentially with growth rate r (i.e., it satisfies the differential equation $y'(t) = ry(t)$). Find a formula for the quantity's doubling time.

7. At 7 AM 100 grams of radioactive material are placed under observation. By 10 AM the amount has decayed to 90 grams.
   (a) Find an equation for the amount of radioactive material t hours after 7 AM.
   (b) What is the half-life of the substance, i.e., how long does it take for the half of the original material to decay?

8. It is reported that Prozac® has a half-life of 2 to 3 days but may be found in your system for several weeks after you stop taking it. What percentage of the original dosage would remain after 2 weeks if the half-life is 2 days? How much would remain if the half-life is 3 days?

Preparation assignment for Friday, 2/8: On Friday we will review for the first midterm, which will be given Monday. Come prepared to take a mock exam during the first hour, and ask questions during the second hour. For your preparation assignment, please turn in the completed class survey distributed in class. (Do NOT sign your survey; I will have you sign something else to record that you did it.)

A little bit of academic rivalry:

Engineers think that their equations are an approximation to reality. Physicists think reality is an approximation to their equations. Mathematicians don't care.

A mathematician, a physicist, and an engineer are travelling through Scotland when they see a black sheep through the window of the train.

"Aha!" says the engineer. "I see that Scottish sheep are black."

"Hmm," says the physicist. "You mean that some Scottish sheep are black."

"No," says the mathematician, "All we know is that there is at least one sheep in Scotland, and that at least one side of that one sheep is black!"

The physicist and the engineer are in a hot air balloon. Soon they find themselves lost in a canyon somewhere. They yell out for help, "Hellllooooooo! Where are we?"

Fifteen minutes later, they hear an echoing voice: "Hellllooooooo! You're in a hot-air balloon!"

The physicist says, "That must have been a mathematician."

"Why do you say that?" the engineer asks.

"Because," replies the physicist, "His answer took a long time to get, it was absolutely correct, and it was utterly useless."