

Selected answers to Merit Worksheet #10

1. The integral $\int_0^1 \frac{1}{x^p} dx$ converges when $p < 1$. The integral $\int_1^\infty \frac{1}{x^p} dx$ converges when $p > 1$.
2. The integral doesn't converge for *any* value of p .
3. (a) $y(t) = -2e^{3t}$
(b) $y(t) = -6e^{-2t}$
(c) $y(t) = 2e \cdot e^{-t} = 2e^{1-t}$
4. (a) $y'(t) = \frac{r}{100}y(t)$, with $y(0) = P$.
(b) $y(t) = Pe^{(r/100)t}$
(c) $\$4500e^{0.08 \cdot 6}$
5. $P(t) = 200e^{[(\ln 3)/5]t} = 200 \cdot 3^{t/5}$. The population will reach 20,000 when $t = 5 \ln(100)/\ln(3)$.
6. Doubling time is $(\ln 2)/r$.
7. (a) $A(t) = 100e^{[\ln(0.9)/3]t} = 100(0.9)^{t/3}$
(b) $t = 3 \ln(1/2)/\ln(0.9)$
8. If the half-life is 2 days, the percentage remaining after 14 days is $e^{\ln(1/2) \cdot 7} = (1/2)^7$. If the half-life is 3 days, the percentage remaining after 14 days is $e^{\ln(1/2) \cdot 14/3} = (1/2)^{14/3}$.