

Merit Worksheet #11, 2/13/08

Infinite sequences

1. In your own words, what is a sequence? (See page 612, if you want, for the book's definition.)
2. In the following, write out the terms a_1, a_2, \dots, a_6 of the given sequence.

$$(a) \quad a_n = \frac{3}{n+4} \quad (b) \quad a_n = 1 + \frac{(-1)^n}{n!} \quad (c) \quad a_n = \sqrt{n} \quad (d) \quad a_n = \frac{(-1)^n n}{n+1}$$

3. Remember Calc I? What did it mean, back then, for

$$\lim_{x \rightarrow a} f(x) = L?$$

What does it mean when we say

$$\lim_{x \rightarrow \infty} f(x) = L?$$

What do you imagine it means, then, to say

$$\lim_{n \rightarrow \infty} a_n = L?$$

4. Find the following limits, if they exist. (If a sequence's limit exists, we say that the sequence **converges** to that number.)

$$(a) \quad \lim_{n \rightarrow \infty} \frac{3}{n+4} \quad (b) \quad \lim_{n \rightarrow \infty} \left(1 + \frac{(-1)^n}{n!}\right) \quad (c) \quad \lim_{n \rightarrow \infty} \sqrt{n} \quad (d) \quad \lim_{n \rightarrow \infty} \frac{(-1)^n n}{n+1}$$

5.

- (a) Compare and contrast $\lim_{x \rightarrow \infty} \sin \pi x$ and $\lim_{n \rightarrow \infty} \sin \pi n$. Indicate the domains of the two functions and how they affect the limits.
- (b) What's the connection, if any, between

$$\lim_{x \rightarrow \infty} f(x) \quad \text{and} \quad \lim_{n \rightarrow \infty} a_n?$$

6. Using limit rules as appropriate, determine whether or not the sequence $\{a_n\}_{n=1}^{\infty}$ converges, and find its limit if it does converge:

$$(a) \quad a_n = \frac{1}{n^3} \quad (b) \quad a_n = \frac{n}{n+1} \quad (c) \quad a_n = 2 - \left(-\frac{1}{2}\right)^n \quad (d) \quad a_n = n \sin\left(\frac{1}{n}\right)$$
$$(e) \quad a_n = \frac{5n^3 - 1}{2n^3 + 1} \quad (f) \quad a_n = (-1)^n \frac{n+4}{n+1} \quad (g) \quad a_n = \frac{e^{2n} + 2}{e^n - 1} \quad (h) \quad a_n = \cos(\pi n)$$

7. Suppose $a_n = n^p$. For which values of p does $\{a_n\}_{n=1}^{\infty}$ converge?
8. Based on our discussion in class, what does it mean for the sequence $\{a_n\}_{n=1}^{\infty}$ to converge to the limit L ? (See also Definition 1.1 on page 613 of your text.)

Given some random ϵ , find N past which the terms of a_n differ from the sequence's limit by at most ϵ :

$$(a) \quad a_n = \frac{1}{n^3} \quad (b) \quad a_n = \frac{n}{n+1}$$

9. (Bonus) Prove that

$$\lim_{n \rightarrow \infty} n^{1/n} = 1.$$

Cool-online-encyclopedia-where-you-can-look-up-whatever-sequence-you-wish of the day:

<http://www.research.att.com/~njas/sequences/>

(My name is buried within the entry for Sequence A000009. Also, be sure to check out the puzzle page at <http://www.research.att.com/~njas/sequences/Spuzzle.html>.)

Quote of the day: “Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.” —Leonhard Euler (1707-1783)