Selected answers to Merit Worksheet #11

2. (a) $\frac{3}{5}, \frac{1}{2}, \frac{3}{7}, \frac{3}{8}, \frac{1}{3}, \frac{3}{10}$
   (b) $0, \frac{3}{2}, \frac{5}{6}, \frac{25}{24}, \frac{119}{120}, \frac{721}{720}$
   (c) $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}$
   (d) $-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \frac{6}{7}$

4. (a) 0
   (b) 1
   (c) The limit does not exist.
   (d) The limit does not exist.

5.
   (a) We find that

$$\lim_{x\to\infty} \sin \pi x \text{ does not exist, while } \lim_{n\to\infty} \sin \pi n = 0.$$

   This is because $\sin \pi x$ has as its domain all real numbers, while $\sin \pi n$ has as its domain only integers. The function $\sin \pi x$ can be seen to oscillate (and thus not converge) when we allow x to be arbitrary real numbers. However, the function $\sin \pi n$ always equals zero when we plug integers in, so that more restrictive domain allows the sequence to converge.

   (b) Assuming that $a_n = f(n)$, we have that if $\lim_{x\to\infty} f(x)$ exists, then $\lim_{n\to\infty} a_n$ also exists, and has the same limit. If $\lim_{x\to\infty} f(x)$ does not exist, then we don't know whether or not $\lim_{n\to\infty} a_n$ exists—it may or may not.

6. (a) Converges to 0.
   (b) Converges to 1.
   (c) Converges to 2.
   (d) Converges to 1.
   (e) Converges to 5/2.
   (f) Does not converge.
   (g) Does not converge.
   (h) Does not converge.

7. Converges for $p \leq 0$.

8. See Definition 1.1 on page 613 of your text. For (a), $N = \sqrt[3]{\frac{1}{\epsilon}}$. For (b), $N = \frac{1}{\epsilon} - 1$.

9. (Hint: Note that

$$\lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty} e^{\ln n^{1/n}} = \lim_{n\to\infty} e^{\frac{1}{n}\ln n} = e^{\lim_{n\to\infty} \frac{\ln n}{n}}.$$

   Then use L'Hospital's Rule.)