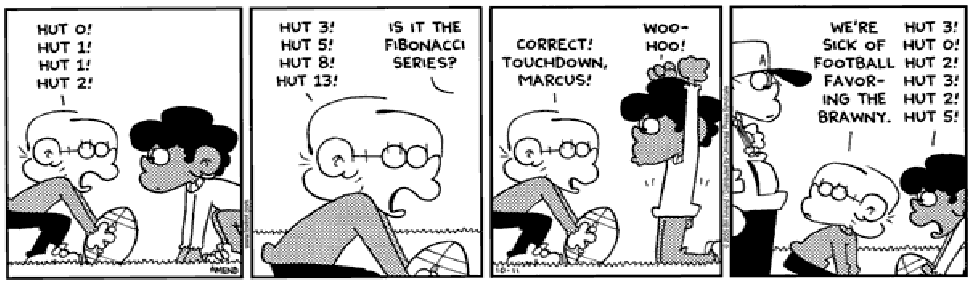
Merit Worksheet #13, 2/18/08

Series

1. What is a series? Give three examples of series. How is a series different from a sequence?

2. Can you find anything mathematically amiss in the following comic strip?[1]



3. What is a *partial sum* of a series? For each of the three series you listed in Problem 1, write out the first four partial sums.

4. What does it mean for a series to converge? Explain how we know that the series in Section 8.2, Examples 2.1 and 2.3 in your text converge, while the series in Example 2.2 diverges.

5. Explain what's illustrated in this table:

n	a_n	S_n
1	1/1	1.00000
2	1/2	1.50000
3	1/6	1.66667
4	1/24	1.70833
5	1/120	1.71667
6	1/720	1.71806
10	$2.756 \cdot 10^{-7}$	1.71828
100	$1.072 \cdot 10^{-158}$	1.71828

   If you had to guess, would you say the series converges or diverges?

6. Let's look at the series $\sum_{n=1}^{\infty} \frac{1}{2^{n+1}}$.

   (a) What is the limit of the sequence $a_n = \frac{1}{2^{n+1}}$?
   (b) Write out the first, second, third, fourth, and fifth partial sums of the series above.
   (c) Based on your answer to part (b), what would you say the series converges to?
   (d) Why is the number you got in part (a) different from the number you got in part (c)?

7. Look now at the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$.

   (a) What is the limit of the sequence $a_n = n/(n+1)$?

[1] *Foxtrot*, by Bill Amend. Reproduced at http://mathworld.wolfram.com/FibonacciNumber.html.

(b) Write out the first, second, third, fourth, and fifth partial sums for the series above. (Use a calculator!)

(c) What do the partial sums you found in part (b) seem to be approaching, if anything?

(d) Does the series above converge? How does this compare with your answer to part (a)?

Special types of series

8. As a group, in unison and with *feeling*, read the first two sentences after the end of Example 2.3 in Section 8.2 of your text.

9. Read the definition of a **geometric series** in the "Note" box on page 629 of your text. Which of the following are geometric series?

$$\sum_{k=0}^{\infty} \frac{1}{3^k} \qquad \sum_{k=4}^{\infty} k^2 \qquad \sum_{k=1}^{\infty} \frac{2^{2k}}{5^{k+1}} \qquad \sum_{k=1}^{\infty} 1 \qquad \sum_{k=0}^{\infty} \frac{2^k - 3^k}{4^k} \qquad \sum_{k=0}^{\infty} \frac{(-1)^k}{k} \qquad \sum_{k=2}^{\infty} \frac{10}{3 \cdot 7^k} \qquad \sum_{k=0}^{\infty} \pi^k$$

10. For each of the geometric series in the last problem, state whether the series is convergent or divergent. If it is convergent, give its value. (See Theorem 2.1 in your text.)

11. True or false: Given *any* geometric series, there's a simple way (requiring fifteen seconds or less) to test whether or not the series converges.

12. Look again at Example 2.3 in your text. Why is that series called a **telescoping sum**? In each of the following series, write the nth partial sum of the infinite series as a telescoping sum and thereby find the sum of the series if it converges:

    (a) $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$ (Hint: simplify your partial sums using natural log rules.)

    (b) $\sum_{k=1}^{\infty} \frac{1}{k(k+2)}$ (Hint: use partial fractions to write the summand in a different way.)

13. What is the **harmonic series?** (See Example 2.7 in your text?) Does the harmonic series converge or diverge?

Review problems

A. Find $\int \frac{1}{x\sqrt{1+x^2}}\,dx$.

B. Find $\int_0^{\infty} \frac{x}{1+x^2}\,dx$.

Anecdote of the day:
The following problem can be solved either the easy way or the hard way:

Two trains 200 miles apart are moving toward each other; each one is going at a speed of 50 miles per hour. A fly starting on the front of one of them flies back and forth between them at a rate of 75 miles per hour. It does this until the trains collide and crush the fly to death. What is the total distance the fly has flown?

The fly actually hits each train an infinite number of times before it gets crushed, and one could solve the problem the hard way with pencil and paper by summing an infinite series of distances.[2] The easy way is as follows: Since the trains are 200 miles apart and each train is going 50 miles an hour, it takes 2 hours for the trains to collide. Therefore the fly was flying for two hours. Since the fly was flying at a rate of 75 miles per hour, the fly must have flown 150 miles. That's all there is to it.

When this problem was posed to the mathematician John von Neumann, he immediately replied, "150 miles."

"It is very strange," said the poser, "but nearly everyone tries to sum the infinite series."

"What do you mean, strange?" asked Von Neumann. "That's how I did it!"

[2] It's actually a geometric series, so while setting up the series is difficult, solving it is easy.