Selected answers for Merit Worksheet #15

1. The series shown is the harmonic series.
   (a) The individual terms converge to 0.
   (b) The series diverges.
2. In part (a), the series adds 2 an infinite number of times, so the sum is infinitely large. In part (b), the series adds terms that are increasingly closer to 1, so it's as if the sum adds 1 an infinite number of times, and therefore is infinitely large. In part (c), the partial sums jump back and forth between 0 and 1, because each additional term either adds or takes away an amount of size 1. In such a situation, there's no hope of the partial sums settling down to a number, so the series diverges.
3. Statement (i) is true; it is the kth-Term Test for Divergence. Statement (ii) is false, as the harmonic series shows.
4. The series in parts (a), (c), (d), (e), and (f) are guaranteed to diverge, because none of them has a term which approaches 0 as k or n grows.
6. The integral test shows that the harmonic series behaves the same way (as far as convergence/divergence go) as the integral $\int_1^\infty (1/x)\,dx$. The integral diverges, so the harmonic series does, too.
7. (a) Diverges, by the integral test. The kth-term test doesn't tell us anything.
   (b) Diverges, by the integral test. The kth-term test doesn't tell us anything.
   (c) Diverges, by the integral test. The kth-term test doesn't tell us anything.
8. A p-series will converge exactly when $p > 1$.
9. The series in (a) will converge, by comparison with the p-series $\sum 1/n^2$. The series in (b) will diverge, by comparison with the harmonic series.
10. The series in (a) will diverge, by the limit comparison test with the harmonic series. The series in (b) will converge, by the limit comparison test with $\sum 1/n^2$.

Review problem
Evaluate the integral $\int x^2 \tan^{-1} x\,dx$.

Oddly enough, this is the same review problem as on Worksheet #12. A complete solution is given in the answers to that worksheet.