

Merit Worksheet #18, 2/29/08

A hodgepodge of series questions

1. So far we've learned about the following tests for the convergence or divergence of a series:

- Test for geometric series
- k th-term test for divergence
- Integral test
- Comparison test
- Limit comparison test
- Alternating series test

Answer the following questions:

- Which of the tests can tell you conclusively that a series converges?
- Which can tell you, conclusively, that a series diverges?
- Which tests require the series terms to be positive?

2. True or false: If

$$\lim_{k \rightarrow \infty} a_k = 0,$$

then $\sum_{k=1}^{\infty} a_k$ converges.

3. Determine if the following series converge or not:

(a) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$

(b) $\sum_{k=0}^{\infty} (-2)^k$

(c) $\sum_{k=0}^{\infty} \frac{k}{k+1}$

(d) $\sum_{k=0}^{\infty} \sin k$

(e) $\sum_{k=1}^{\infty} \frac{k}{k^3+1}$

(f) $\sum_{k=1}^{\infty} \left(\frac{5}{2}\right)^{-k}$

(g) $\sum_{k=0}^{\infty} \frac{\tan^{-1} k}{1+k^2}$

(h) $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

(i) $\sum_{k=1}^{\infty} \frac{\ln \sqrt{k}}{k}$

(j) $\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

(k) $\sum_{k=1}^{\infty} \frac{k + \sqrt{k}}{2k^3 - \sqrt{k}}$

(l) $\sum_{k=1}^{\infty} \frac{k}{2^k}$

(m) $\sum_{k=1}^{\infty} \sin(\pi/k)$

(n) $\sum_{k=1}^{\infty} (-1)^k \frac{\cos \pi k}{k}$

(o) $\sum_{k=1}^{\infty} (-1)^k \frac{k}{k^2+1}$

(p) $\sum_{k=0}^{\infty} \sin\left(\frac{k\pi}{4}\right)$

$$(q) \sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k}$$

$$(r) \sum_{k=1}^{\infty} \left(\frac{k+1}{k} \right)^k$$

$$(s) \sum_{k=2}^{\infty} \frac{3}{10^k}$$

$$(t) \sum_{k=3}^{\infty} \frac{1}{k^2 - k}$$

4. Give a proof, using geometric series, that

$$0.99999\dots = 1.$$

5. Find the *sums* of the series in Problem 3, parts (a), (f), (s), and (t).

6. Find the sum of the series in Problem 3(1). Your book describes one method for doing so in the exercises for Section 8.3.

Preparation for next time: Read Section 8.5 through Theorem 5.1, and read Example 5.3. Read the boxes containing the Ratio Test (page 658) and the Root Test (page 661), and read Examples 5.4 through 5.8. Prepare Exercise 11 to turn in.