Merit Worksheet #19, 3/3/08

Absolute convergence

1. What does it mean for a series to converge absolutely? Conditionally? (See pg. 656 of your text.)
2. Which of the following series converge absolutely? Conditionally? Not at all?

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \qquad \sum_{k=1}^{\infty} \frac{1}{k} \qquad \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

3. Now look again at Theorem 5.1 on page 657. What does it mean? Can you think of a real-life analogy? (Think along the lines of "You'll never know (if you can reach infinity) unless you try," or something similar.

The two tests we'll cover today—the ratio test and the root test—tell us whether or not a series converges *absolutely.*

Geometric series again, really quickly

4. Under what circumstance does the geometric series $\sum_{k=0}^{\infty} ar^k$ converge?

The ratio test

5. Review the ratio test, presented on page 658 of your text. Then use the ratio test to decide whether or not the following series converge:

$$\text{(a)} \quad \sum_{k=1}^{\infty} \frac{(-1)^k k}{3^k} \qquad \text{(b)} \quad \sum_{k=1}^{\infty} \frac{10^n}{n!} \qquad \text{(c)} \quad \sum_{n=1}^{\infty} \frac{e^n}{n}$$

6. For each of the geometric series below, what's the ratio a_{k+1}/a_k?

$$\text{(a)} \quad \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k \qquad \text{(b)} \quad \sum_{k=1}^{\infty} 10 \cdot 3^k \qquad \text{(c)} \quad \sum_{k=0}^{\infty} ar^k$$

7. How might the ratio test show you how much a series behaves like a geometric series?
8. What does the ratio test tell you about each of the following series?

$$\text{(a)} \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \qquad \text{(b)} \quad \sum_{k=1}^{\infty} \frac{1}{n^2+1} \quad \text{(c)} \quad \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

The root test

9. Quick! As a bit of preparation for the next problems, find the following:

$$\text{(a)} \quad \lim_{k\to\infty} 2^{1/k} \qquad \text{(b)} \quad \lim_{k\to\infty} k^{1/k} \qquad \text{(c)} \quad \lim_{k\to\infty} \left(k^2\right)^{1/k}$$

10. Review the root test, stated on page 661 of your text. Then use the root test to determine whether or not the series below converge:

$$\text{(a)} \quad \sum_{k=1}^{\infty} \frac{(-1)^k k}{3^k} \qquad \text{(b)} \quad \sum_{k=1}^{\infty} \left(\frac{3k-1}{4k+6}\right)^k \qquad \text{(c)} \quad \sum_{n=1}^{\infty} \frac{e^n}{n}$$

11. For each of the geometric series below, what's the limit $\lim_{k\to\infty} (a_k)^{1/k}$?

$$\text{(a)} \quad \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k \qquad \text{(b)} \quad \sum_{k=1}^{\infty} 10 \cdot 3^k \qquad \text{(c)} \quad \sum_{k=0}^{\infty} ar^k$$

12. How might the root test show you how much a series behaves like a geometric series?

13. What does the root test tell you about each of the following series?

$$\text{(a)} \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \qquad \text{(b)} \quad \sum_{k=1}^{\infty} \frac{1}{n^2+1} \qquad \text{(c)} \quad \sum_{k=1}^{\infty} \frac{k}{2^k}$$

A bonus problem

14. The *Fibonacci sequence* is the sequence F_k where $F_1 = 1$, $F_2 = 1$, and $F_k = F_{k-2} + F_{k-1}$ for every $k \geq 3$. (In words, each term is the sum of the two previous terms.) The first terms of the Fibonacci sequence are listed below:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, \ldots$$

The Fibonacci sequence has many remarkable properties. One is that it behaves almost like a geometric sequence. If you look at the ratios

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}, \frac{144}{89}, \frac{233}{144}, \frac{377}{233}, \frac{610}{377}, \frac{987}{610}, \ldots,$$

you'll see that they seem to approach a single number, approximately 1.618.

(a) Suppose the limit F_{k+1}/F_k of consecutive terms is L. Working with the equation

$$F_k = F_{k-2} + F_{k-1}, \qquad \text{we get} \qquad \frac{F_k}{F_{k-1}} = \frac{F_{k-2}}{F_{k-1}} + 1.$$

What should the exact value of L be?

(b) Use the ratio test to determine whether or not the series $\sum_{k=1}^{\infty} \frac{1}{F_k}$ converges.

(c) Assuming that $F_k \approx L^k$, where L is the limit you found in part (a), use the root test to determine whether or not the series $\sum_{k=1}^{\infty} \frac{1}{F_k}$ converges.

Preparation for next time, and a stupid math joke

Next time we'll review all the series convergence tests. There will be nothing to turn in for next time, but in order to be ready, you should complete all of the (non-bonus) problems on today's worksheet.

Two mathematicians are studying a convergent series.
The first one says, "Do you realize that the series converges even when all the terms are made positive?"
The second one gasps, "Are you sure?"
"Absolutely!"