Merit Worksheet #33, 4/21/08

Introduction to polar coordinates

1. Plot the point given in polar coordinates, and find the corresponding cartesian coordinates for the point:

   (a) $\left(4, \frac{\pi}{2}\right)$ (b) $\left(-1, \frac{5\pi}{4}\right)$ (c) $\left(\frac{3}{2}, \frac{5\pi}{2}\right)$

2. In the problems below the rectangular coordinates of a point are given. Plot the point and find two sets of polar coordinates for the point (where your coordinates satisfy $0 \leq \theta < 2\pi$):

   (a) $(1, 1)$ (b) $(-6, 0)$ (c) $(5, 12)$

3. Express the given rectangular equations in polar form:

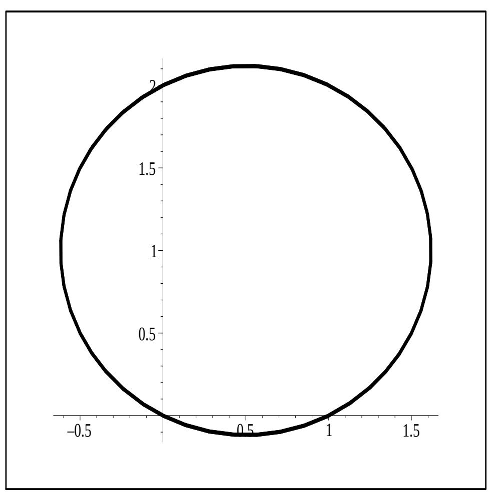
   (a) $x^2+y^2-4x = 0$ (b) $3x-y+2 = 0$ (c) $xy = 4$ (d) $(x^2+y^2)^2-9(x^2-y^2) = 0$

4. Express the given polar equations in rectangular form:

   (a) $r = 4$ (b) $r = 4\sin\theta$ (c) $\theta = \pi/6$ (d) $r^2 = \sin 2\theta$

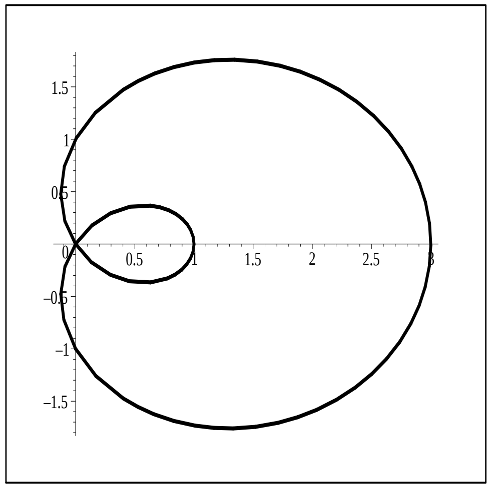
5. Write equations in both rectangular and polar forms for the line with x-intercept A and y-intercept B.

6. Which of the polar equations below describes the following graph?



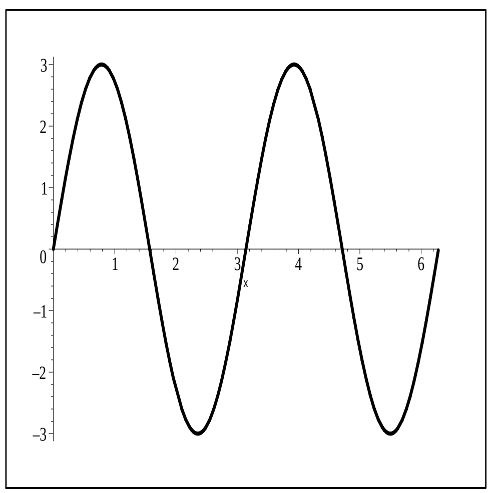
(i) $r = 1$ (ii) $r = \frac{\sqrt{5}}{2}$ (iii) $r = 1 + \sin\theta$ (iv) $r = \cos\theta + 2\sin\theta$

7. Which of the polar equations below describes the following graph?



(i) $r = -1 + 2\cos\theta$ (ii) $r = 3 - 2\sin\theta$ (iii) $r = 2 + \cos\theta$ (iv) $r^2 = 9\cos\theta$

8. Shown here is a graph of $y = 3\sin 2x$:



Use the information contained in this graph to help you plot the polar curve $r = 3\sin 2\theta$.

9. (a) Plot the graph $y = 1 + \cos x$ between $x = 0$ and $x = 2\pi$ on a pair of x, y-axes.
   (b) In your drawing for part (a), erase the "x" label on the horizontal axis, and write θ in its place. Erase the "y" labelling the vertical axis, and write r in its place.
   (c) Using the graph you now have to aid you, graph the polar equation $r = 1 + \cos\theta$.

10. Find all points of intersection of the two polar curves:
    (a) $r = 1$, $r = \sin\theta$
    (b) $r = 1 + \cos\theta$, $r = 1 - \sin\theta$

11. Test the polar equation $r = 2\sin 5\theta$ for symmetry about the x- or y-axes, or about the origin.

Another very bad pun

Q: Whats a polar bear?
A: A rectangular bear after a coordinate transform.

For next time

Next time we will cover all of Section 9.5 (Calculus and Polar Coordinates). Please read the beginning of the section through Example 5.1. Then read the boxed area formula on page 758, together with Examples 5.3, 5.4, and 5.5. Then read the boxed arc length formula on page 762, together with Example 5.8. Prepare Exercises 1, 15, and 35.

Watching *High School Musical* makes you smarter!

Friday and Saturday I attended a math conference at Illinois State University. One of my favorite talks was by Professor Bruce C. Berndt (from our very own department—take a class from him, if you can) about the famous Indian mathematician Ramanujan and series he discovered which converge to $1/pi$. Two of these series can be written as

$$\frac{4}{\pi} = \sum_{k=0}^{\infty} \frac{(6k+1)}{4^k} \left[\frac{(1/2)(1/2+1)(1/2+2)\dots(1/2+k-1)}{k!} \right]^3$$

and

$$\frac{16}{\pi} = \sum_{k=0}^{\infty} \frac{(42k+5)}{2^{6k}} \left[\frac{(1/2)(1/2+1)(1/2+2)\dots(1/2+k-1)}{k!} \right]^3 .$$

Using a bit of math notation not commonly taught in high school, and changing the k's to n's, we can write that last one more concisely as

$$\frac{16}{\pi} = \sum_{n=0}^{\infty} \frac{(42n+5)\left(\frac{1}{2}\right)_n^3}{64^n (n!)^3}.$$

Professor Berndt pointed out that if you've watched Disney's *High School Musical*, then you've seen this formula before! There's a scene where Gabriella Montez corrects a formula her teacher's written on the board, prompting her to change an 8 into a 16 in the formula above. If you'd like to see for yourself, you can visit a blog which points this out (and links to a clip from the movie) at `http://unimodular.net/blog/?p=116`. I have to agree with Sharpay's open-mouthed reaction to Gabriella's catch, but I have to wonder (a) why the math class is being taught in a chemistry lab, and (b) why the high school class is covering graduate-level analytic number theory.

Amazing, right? See why I go to math conferences?

Review Problems for the Final — Sections 8.4 and 8.5

These problems are provided in preparation for your final. They are typical of what you can expect from Sections 8.4 and 8.5. As before, in an effort to motivate you to work with your classmates and not postpone thinking about these problems, I will *not* be posting solutions to these problems. If you get stuck in working a problem, let me or a fellow class member help you out. Good luck!

Determine whether the following series converge absolutely, converge conditionally, or diverge.

A. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{3}{k!}$ B. $\displaystyle\sum_{k=0}^{\infty}(-1)^k 2^k$ C. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{2}{3^k}$

D. $\displaystyle\sum_{k=1}^{\infty}\frac{k}{k^2+1}$ E. $\displaystyle\sum_{k=0}^{\infty}\frac{3^k}{k!}$ F. $\displaystyle\sum_{k=3}^{\infty}(-1)^{k+1}\frac{4}{2k+1}$

G. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{4k+1}{5k-7}\right)^k$ H. $\displaystyle\sum_{k=1}^{\infty}\frac{e^{3k}}{k^{3k}}$ I. $\displaystyle\sum_{k=1}^{\infty}\frac{\cos k}{k^3}$

How big could the error be between the tenth partial sum and actual series value for the following?

J. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{4}{k^3}$ K. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2}{k}$ L. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}e^{-k}$