Name: _____

# Math 231 W3, Spring Term 2009
# Mock Exam #3

April 16, 2009

No books, notes, calculators, or other aids may be used. For full credit you must show all your work on each problem.

| Problem | Score | Points Possible |
|---|---|---|
| 1 | | 17 |
| 2 | | 17 |
| 3 | | 17 |
| 4 | | 17 |
| 5 | | 16 |
| 6 | | 16 |
| TOTAL | | 100 |

**Problem 1: (17 points) (a) Find the fourth-degree Taylor polynomial $P_4(x)$ for the function $1/x$, expanded about the point $c = 10$.**

**(b) Write down the remainder term (error term) $R_4(x)$ that goes with this polynomial. Be sure to state which quantities $z$ lies between.**

**(b) If we used $P_4(7)$ as an approximation for $1/7$, how big could the error be?**

**Problem 2: (17 points) (a) The Taylor series for the function** $f(x) = \ln\left(\frac{x+1}{2}\right)$ **is**

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k \cdot k}(x-1)^k.$$

**Find both the <u>interval</u> and the <u>radius</u> of convergence of this series.**

**(b) May we use the Taylor series to approximate** $\ln\left(\frac{1+1}{2}\right)$, $\ln\left(\frac{1+3}{2}\right)$, **or** $\ln\left(\frac{1+5}{2}\right)$**? Explain your answer for each the three values.**

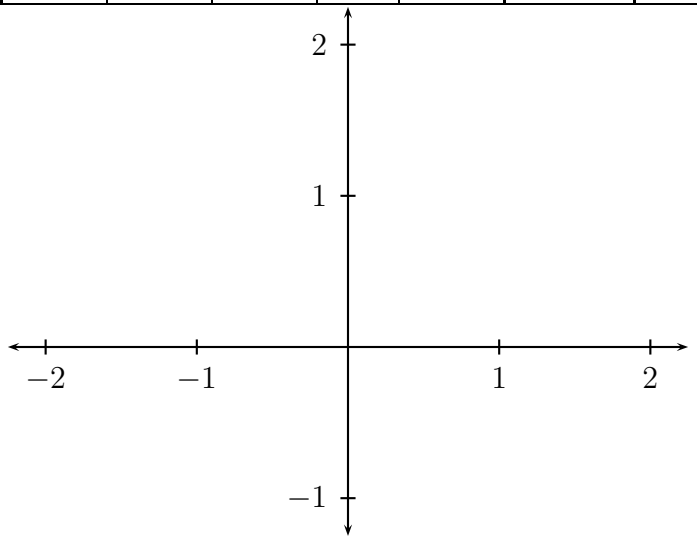
**Problem 3: (17 points) Use four terms of a known power series to approximate** $\int_0^1 e^{-x^2}\,dx$.

**Problem 4: (17 points) Consider the curve $x = \cos^2 t$, $y = 1 - \sin t$.**

**(a) Eliminate the parameter $t$ to get an equation just in terms of $x$ and $y$.**

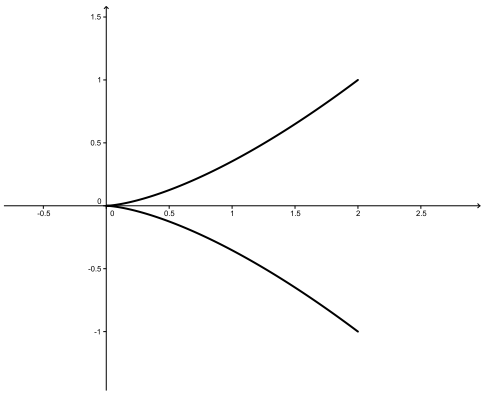
**(b) Make a rough sketch of this curve when $0 \leq t \leq \pi$. Use the axes below, and consult the table of trig values as necessary.**

| $\theta$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $3\pi/4$ | $5\pi/6$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | 0 |
| $\cos\theta$ | 1 | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | 0 | $-1/2$ | $-\sqrt{2}/2$ | $-\sqrt{3}/2$ | $-1$ |




**(c) Describe in words how the curve is traced out (eg., where the curve starts when $t = 0$, where it heads next, and where it ends up at $t = \pi$).**

**Problem 5: (16 points) The parametric curve $x = 2t^2$, $y = t^3$, $-1 \leq t \leq 1$ is sketched below:**




**(a) Find the length of the curve over the interval $-1 \leq t \leq 1$.**

**(b) Find the area between the curve and the $x$-axis for the portion of the curve traced out as $0 \leq t \leq 1$.**

**Problem 6: (16 points) (a) Find the arc length of the curve** $x = t$ **,**$y = \frac{2}{3}(t-1)^{3/2}$ **from** $t = 1$ **to** $t = 5$**.**

**(b) Find the area of the surface of revolution when the curve** $x = t$**,** $y = t^3$**,** $1 \leq t \leq 2$**, is revolved around the** $x$**-axis.**