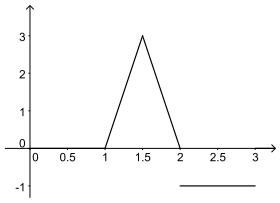
Merit Worksheet #1, 1/21/09

Let's talk about integrals. As a group, answer the following.

1. Shown below is a picture of $f(x)$ on the interval $[1, 3]$.



   (a) Conceptually, what exactly does $\int_1^3 f(x)\,dx$ represent?

   (b) What would $\int_1^3 f(x)\,dx$ be, in this case?

2. Find the following quantities.

   (a) The area between $y = x^2$ and the x-axis on the interval $[0, 1]$.

   (b) $\displaystyle\int_{-2}^{-1} \frac{1}{x}\,dx + \int_0^1 \frac{1}{1+x^2}\,dx - \int_0^{\pi/2} \sin x\,dx.$

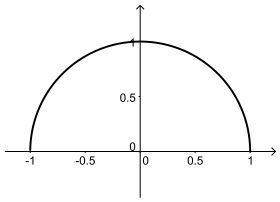
   (c) $\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}\,dx + \int_0^1 e^x\,dx.$

3. (a) Show that $F_1(x) = 1/(1-x)$ is an antiderivative of $f(x) = 1/(1-x)^2$.

   (b) Show that $F_1(x) = x/(1-x)$ is also an antiderivative of $f(x) = 1/(1-x)^2$.

   (c) How is it possible that these two *different* functions are both antiderivatives of $f(x)$? What, if any, is the relationship between the two?

4. Shown below is the graph $y = \sqrt{1-x^2}$ on the interval $[-1, 1]$.



   (a) What's $\displaystyle\int_{-1}^1 \sqrt{1-x^2}\,dx$?

   (b) What makes finding $\displaystyle\int_{-1}^{1/2} \sqrt{1-x^2}\,dx$ tricky (for you, who've only seen Calculus I)?

   (c) Using what you learned in Calc I, how could you approximate $\displaystyle\int_{-1}^{1/2} \sqrt{1-x^2}\,dx$?

   (d) Wonder aloud (with feeling) whether it's possible that you'll learn techniques in Calc II allowing you to find $\int_a^b \sqrt{1-x^2}\,dx$ for *any* a and b?

   (e) Throw calculus aside and use geometry (and maybe some trigonometry) to calculate $\displaystyle\int_{-1}^{1/2} \sqrt{1-x^2}dx$.

5. Find the following antiderivatives.

(a) $\displaystyle\int \left(e^{3\pi^x} + \frac{2}{3x}\right) dx$ (b) $\displaystyle\int (1 + \sec 4x \tan 4x)\, dx$ (c) $\displaystyle\int \cos x e^{\sin x}\, dx$ (d) $\displaystyle\int \frac{x^2}{1+x^6}\, dx$

(e) $\displaystyle\int \frac{1}{\sqrt{12x - 4x^2 - 8}}\, dx$

Quote of the day:
"Do not worry about your difficulties in mathematics. I can assure you mine are still greater."
—Albert Einstein (1879 - 1955)