Merit Worksheet #2, 1/23/09

Integration by Parts

1. Look at the integral
$$\int x \cos x \, dx.$$
It's tempting, I know, but at all costs ***DON'T*** write
$$\int x \cos x \, dx = \frac{1}{2}x^2 \sin x + C.$$
   (a) Can you explain why someone might write the equation above?
   (b) Can you show why it's completely, utterly, horribly wrong?
   (c) Can you draw an analogy with the problem above and the product rule of derivatives?

2. Let's take another look at the integral
$$\int x \cos x \, dx.$$
Say you decide to try to find this integral through integration by parts. (Good thinking, by the way.)
   (a) Write down at least three possible choices for u and dv.
   (b) For each choice of u and dv you came up with in part (a), what does the integration by parts formula (the top box on page 515 of your text) give you?
   (c) Choose the best-looking answer to part (b), and find the answer to the integral.

3. Find the following integrals using integration by parts:
   (a) $\int xe^{2x} \, dx$ (b) $\int \arctan 3x \, dx$

4. Why is it *never* a good idea, when applying integration by parts to $\int f(x) \, dx$, to let $u = 1$ and $dv = f(x) \, dx$?

5. Find the following integrals by integration by parts:
   (a) $\int x \sin 2x \, dx$
   (b) $\int_1^e x^2 \ln x \, dx$
   (c) $\int_0^2 t^2 e^{5t} \, dt$
   (d) $\int_0^1 x \arcsin(x^2) \, dx$

6. Find the integral
$$\int e^{3x} \cos 2x \, dx.$$
(Hint: you'll find it helpful to integrate by parts *twice*. Pay close attention to what you arrive at the second time around.)

7. Look at the second integration by parts you did in Problem 5. What would happen if you did it differently by switching at this point what u and dv are? Why does this happen? Concoct a moral to this story, maybe having something to do with "switching horses midstream" or something of that nature.

8. Opening up a calculus book at random, you see the *reduction formula*

$$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx.$$

   (a) Use the formula to find $\int x^3 e^x \, dx$.
   (b) How did the book derive this formula?

9. Look at the integral

$$\int_{-1}^{2} y\sqrt{y+2} \, dy.$$

   a) Find this integral by integration by parts.
   b) Find this integral through a u-substitution.
   c) Are your answers to (a) and (b) equal?

10. Find the integral

$$\int e^{-\sqrt{x}} \, dx.$$

   (Hint: make a substitution first, and use integration by parts on the result.)

Reading assignment for Monday, 1/26: Next time we'll cover the material on pages 521 to 525 of your text (note that this is only half of Section 6.3). Read with attention all the text of the section through Example 3.1. Make sense of Examples 3.4 and 3.6 (and read the paragraphs just before each one). Skim the rest of the material. Write up an answer to problem 1 in the exercises (*not* Writing Exercise 1). Make sure to show *all* your steps, and turn that and your reading question in on Monday.

Proofs to impress your teachers and amaze your friends: Let's apply integration by parts to the integral

$$\int \frac{1}{x} \, dx,^*$$

with $u = 1/x$ and $dv = 1 \, dx$. We get

$$\int \frac{1}{x} \, dx = \frac{1}{x} \cdot x - \int x \cdot -\frac{1}{x^2} \, dx = 1 + \int \frac{1}{x} \, dx.$$

Subtracting $\displaystyle\int \frac{1}{x} \, dx$ from both sides, we find that

$$\boxed{0 = 1}.$$

Any questions?

*Yes, I know that we don't *need* to use integration by parts to get the answer. Just humor me for a bit.