Merit Worksheet #3, 1/26/09

Trigonometric integrals

u-substitutions

1. Evaluate the following integrals by u-substitution:

(a) $\int \cos^2 x \sin x \, dx$ (b) $\int \cos^3 x \sin^4 x \, dx$ (c) $\int \sin^3 x \, dx$ (d) $\int \tan x \, dx$

(e) $\int \tan x \sec^2 x \, dx$ (f) $\int \tan^3 x \sec x \, dx$ (g) $\int \tan^4 x \sec^2 x \, dx$

2. What is it about the integrals in Problem 1 might suggest to you that u-substitution would help?

Identities and integration by parts

3. We wish to find the integral $\int \cos^2 x \, dx$.

   a) Use the identity $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ to find the integral.

   b) Can you find the integral by using integration by parts (and then an identity)?

4. Find the integral $\int \sin^2 x \cos^2 x \, dx$.

5. (a) What is $\int \sec x \, dx$? (This one's tricky! See Example 3.8 on page 525 of your text.)

   (b) Find $\int \sec^3 x \, dx$.

6. Consider the following integral:

$$\int \sin 2x \cos 3x \, dx$$

   (a) Perhaps the easiest way to do this integral is to use the handy trigonometric identity

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

   Using this identity, evaluate the antiderivative above.

   (b) Let's suppose that, like most people, you didn't have the identity from part (a) memorized. Find the integral in a way that doesn't use any trig identities.

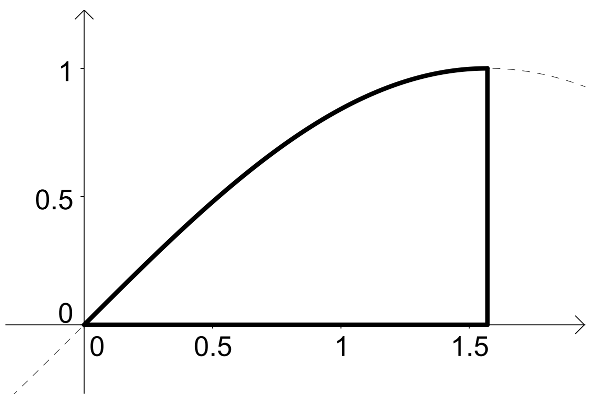
7. When, would you say, is it a good idea to use integration by parts and/or trigonometric identities to find an antiderivative?

An application

In first-semester calculus you may have learned that the *centroid* of a region is the location of its "center of gravity," i.e. if you wanted to balance the region on a pencil, it's where you would stick the point. The coordinates of the centroid are denoted by $(\overline{x}, \overline{y})$, and for the centroid of the region bounded by $y = f(x)$, the x-axis, $x = a$ and $x = b$ are given by

$$\overline{x} = \frac{\int_a^b x f(x)\, dx}{\int_a^b f(x)\, dx} \qquad \text{and} \qquad \overline{y} = \frac{\frac{1}{2}\int_a^b f(x)^2\, dx}{\int_a^b f(x)\, dx}.$$

8. Find the centroid (both coordinates) of the region enclosed by the x-axis, the line $x = \pi/2$, and the graph $y = \sin x$.



Looking at the graph above, does your answer make sense?

Trig facts you'll need to have memorized:

$$\sin^2\theta + \cos^2\theta = 1; \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}; \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}; \qquad 1 + \tan^2\theta = \sec^2\theta.$$

In addition, you'll need to know the derivatives of $\sin x$, $\cos x$, $\tan x$, and $\sec x$, and the antiderivatives $\int \sin x\, dx$, $\int \cos x\, dx$, and $\int \sec x\, dx$.

Reading assignment for Wednesday, 1/23: Next time we'll finish the material in Section 6.3. Carefully read all of the section beginning with the heading "Trigonometric substitution" on page 525. Make sure you understand the three examples. Prepare the solution to Exercise 17 to turn in; make sure to show *all* your steps. Also, don't forget to write down a question you have after doing the reading.

Quote of the day: "But just as much as it is easy to find the differential of a given quantity, so it is difficult to find the integral of a given differential. Moreover, sometimes we cannot say with certainty whether the integral of a given quantity can be found or not." — Johann Bernoulli, 1667-1748