Merit Worksheet #7, 2/6/09

A bunch of integral problems

1. Evaluate each of the following antiderivatives *in at least two separate ways:*

   (a) $\displaystyle\int x\sqrt{1-x}\,dx$ (b) $\displaystyle\int \frac{1}{x(\ln x)^3}\,dx$ (assume $x>0$) (c) $\displaystyle\int \frac{\ln x}{x}\,dx$

   (d) $\displaystyle\int \sin x\cos x\,dx$ (e) $\displaystyle\int \frac{x}{9+x^2}\,dx$

2. So you think the integral $\displaystyle\int \sec x\,dx$ comes out of nowhere? Let's take another look at it.

   (a) Write down (from memory) what the antiderivative is.
   (b) Derive the antiderivative again, starting by writing $\sec x$ as $1/\cos x$ in the integral, and then multiplying both numerator and denominator by $\cos x$.
   (c) Are the answers you got in parts (a) and (b) equivalent?

3. Find the following integrals (the starred problems may or may not be a bit more of a challenge):

   (a) $\displaystyle\int_0^1 \frac{x^3}{1+x^4}\,dx$ (b) $\displaystyle\int \sec^4 x\,dx$ (c) $\displaystyle\int \tan^{-1} x\,dx$ (d) $\displaystyle\int \frac{x^2+3}{x^2-3x+2}\,dx$

4. Find the partial fraction decomposition of the rational functions

   (a) $r(x)=\dfrac{x^2-3x-1}{x^3+x^2-2x}$ (b) $r(x)=\dfrac{x^4}{(x-1)^3}$

Watching the Superbowl makes you smarter!

It's been almost a week, but who can forget it?

5. To commemorate the Steelers' win in the Superbowl, suppose we wanted to make our very own Vince Lombardi trophy out of sterling silver (or, to be more economical, milk chocolate) by revolving the curve $y=\sin x$ between $x=0$ and $x=\pi$ about the x-axis.

   (a) Referring to Section 5.2 of your text as necessary (page 446), find the volume of such a football.
   (b) Referring to Section 5.4 of your text as necessary (page 470), find the surface area of the football.

6. According to Wikipedia, the shape used in the Steelers logo is called an *astroid*, and it has the equation $x^{2/3}+y^{2/3}=1$.

   (a) Find the area enclosed by one astroid.
   (b) Referring again to Section 5.4 (page 466), find the perimeter of one of the astroids.

And back to some more integrals

7. Evaluate the given integrals:

   (a) $\displaystyle\int \frac{1}{t([\ln t]^2 - 4)}\, dt$

   (b) $\displaystyle\int \frac{\sec^2\theta}{\tan^3\theta - \tan^2\theta}\, d\theta$

   (c) $\displaystyle\int \frac{dt}{e^t + 5 + 6e^{-t}}$

Reading assignment for Monday, 2/9: On Monday we'll cover the first half of Section 6.6 on improper integrals. Read the section from the beginning through the end of Example 6.7 on page 552. Skim the rest of the section through Example 6.10, and prepare Exercise 1 and a written question to turn in.

Joke of the day: —*Foxtrot*, by Bill Amend (accessed at `http://www.te-mat.org/Essays/kouba.aspx`)