Merit Worksheet #16, 3/4/08

Alternating series

1. Which of the following tests only work apply to series whose terms are *positive*?

Test for geometric series (pg. 629)	kth-Term Test (pg. 632)	Integral Test (pg. 638)
Rule for p-series (pg. 639)	Comparison Test (pg. 641)	Limit Comparison Test (pg. 643)

2. Quick review: What is a series, what's a partial sum, and what does it mean for a series to converge?

3. (a) Look up the definition of an alternating series on page 648 of your text. True or false: when the book talks about the series terms and writes a_k, like in that equation at the bottom of the page, a_k is a *positive* number.

   (b) Now look up the Alternating Series Test on page 649. What must be true of an alternating series in order for you to conclude from this test that it converges?

   (c) Have someone from your text get up and give a visual explanation on the chalkboard for why the Alternating Series Test works.

   (d) Can you ever conclude from the Alternating Series Test that a series *diverges*?

   (e) What other series test have we learned that can't be used to show both convergence and divergence?

4. Which of the following series converges?

   (a) $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{2}{k^2}$ (b) $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k+1}$ (c) $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{2^k}$ (d) $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{4^k}{k!}$

   Remember in each case to check *all* of the conditions you listed in Problem 1, part (b). If the Alternating Series Test doesn't work on a series, use some other convergence/divergence test.

5. Quick review: True or false? If $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k\to\infty} a_k = 0$.

6. (a) What does the geometric series $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k}$ converge to?

   (b) List (and simplify) the first ten partial sums of the series.

   (c) Sketch a number line, and mark on it the location of each of the partial sums and the location of the limit of the series. What patterns do you see in your graph? Can you explain why they happen?

7. (a) Suppose I want to approximate the series $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2}{k^3}$, so I compute the 10th partial sum. How close can I be sure that my estimate is to the series' sum?

   (b) What if I compute the 100th partial sum—how close is my estimate then?

   (c) How many terms would I need to include to ensure that my partial sum is within 1/1000 of the series value?

8. Let's play around some more with the harmonic series.

   (a) Does the harmonic series converge or diverge?

(b) How about the alternating harmonic series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$?

(c) Let's pull out just the positive terms of the alternating harmonic series—does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ converge?

(d) And now for the negative terms: does the series $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{2n}$ converge?

(e) What conclusions, if any, do you draw from comparing your answers to parts (a) through (d) above?

9. Wonder why the requirement that $a_{k+1} \leq a_k$ is part of the Alternating Series Test? We take the following exercise from your textbook:

   Let

$$a_k = \begin{cases} 1/k & \text{if } k \text{ is odd,} \\ 1/k^2 & \text{if } k \text{ is even.} \end{cases}$$

   Show that $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ diverges, even though it's an alternating series with $\displaystyle\lim_{k\to\infty} a_k = 0$.

10. Here's another problem from your text: A person starts walking from home (at $x = 0$) toward a friend's house (at $x = 1$). Three-fourths of the way there, he changes his mind and starts walking back home. Three-fourths of the way home, he changes his mind again and starts walking back to his friend's house. Three-fourths of the way back to the last spot he turned around, he turns back again. If he continues this pattern of indecision, always turning around at three-fourths of the distance to the last spot he turned around, what will be the eventual outcome?

Reading assignment for next time

For Friday please read Section 8.5 through Theorem 5.1, and read Example 5.3, the paragraph after it, and the box containing the Ratio Test (page 658). Then read Examples 5.4 through 5.7. Prepare Exercise 11 to turn in on Friday, along with a reading question. The Ratio Test may or may not seem pretty straightforward to you, so feel free to get creative with your questions.

Math joke of the day

"So how's your boyfriend doing, the math student?"

"Don't mention that crazy moron to me anymore! We broke up."

"What did he do? He seemed like such a nice guy."

"Ugh! He was restless during the days and couldn't sleep at night—always trying to solve his math problems. When he had finally done it, he wasn't happy: he would call himself a complete idiot and throw all his notes into the garbage. One day, I couldn't take it anymore, and I told him to drop math. You know what he told me?"

"What?"

"He said he enjoyed it!"