Merit Worksheet #21, 3/18/09

Power series

1. Which of the following series converge?

   (a) $\displaystyle\sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k$ (b) $\displaystyle\sum_{k=0}^{\infty}3^k$ (c) $\displaystyle\sum_{k=0}^{\infty}(0.999)^k$ (d) $\displaystyle\sum_{k=0}^{\infty}(-1)^k$

2. Now look at the series $\displaystyle\sum_{k=0}^{\infty}x^k$. If I asked you whether or not this series converged, why would you have to say, "It depends"? What does it depend upon?

3. Which of the following series converge?

   (a) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\left(-\frac{1}{3}+1\right)^k$ (b) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(-1+1)^k$ (c) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(1.2+1)^k$

   (d) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(-2+1)^k$ (e) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(0+1)^k$

4. For which x does the series $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(x+1)^k$ converge? (Hint: use the Ratio Test as a starting point.)

5. For which x does the series $\displaystyle\sum_{k=1}^{\infty}k!(x-2)^k$ converge? (Hint: again, begin with the ratio test.)

6. What exactly *is* a power series? (See page 665 of your text.)

Look at your answers to Problems 1 through 5. It turns out that the collection of x's which make a power series converge always forms an *interval.* Did your answers to Problems 2, 4, and 5 involve intervals?

7. Define the interval of convergence and the radius of convergence for a series. (See the top paragraph on page 666 of your text.) For the series in problems 2, 4, and 5, what are the intervals and radii of convergence?

8. (a) True or false: the ratio test alone will tell you what the radius of convergence is for a power series.
   (b) Why will the ratio test never tell you about the convergence or divergence of a series at the endpoints of the interval of convergence?

9. Find the interval and radius of convergence:

   (a) $\displaystyle\sum_{k=0}^{\infty}\frac{k}{6^k}(x-3)^k$ (b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2}x^k$ (c) $\displaystyle\sum_{k=0}^{\infty}\frac{x^k}{k!}$ (d) $\displaystyle\sum_{k=1}^{\infty}\frac{\ln k}{k}(x+1)^k$

10. Suppose that the power series $\displaystyle\sum_{k=0}^{\infty}a_k(x-1)^k$ converges when $x=3$. What can you say about the convergence or divergence of the following?

    (a) $\displaystyle\sum_{k=0}^{\infty}a_k$ (b) $\displaystyle\sum_{k=0}^{\infty}(-1)^k a_k$ (c) $\displaystyle\sum_{k=0}^{\infty}(-1)^k a_k 2^k$

11. Suppose that the power series $\sum_{k=0}^{\infty} a_k(x+2)^k$ converges at $x = 4$. At what other values of x must $\sum_{k=0}^{\infty} a_k(x+2)^k$ converge?

12. And here's a hint of Friday's stuff: Using what you've learned, you can find the interval of convergence for the power series $\sum_{k=0}^{\infty}(-x)^k$. But can you write down a formula for what the series converges *to*? And once you've done that, can you figure out a power series that might converge to $\ln(1+x)$?

Preparation for next time

On Friday we will have a quiz covering the material on this worksheet, and we will also talk some more about power series and how they relate to functions (and why they're awesome). In preparation for that, please read the paragraphs between Examples 6.4 and 6.5 on pages 667 through 668 of your text, and Example 6.6 through the end of the section. Prepare and turn in a reading question and an answer to the following:

The geometric series formula shows that

$$\frac{a}{1-x} = \sum_{k=0}^{\infty} ax^k$$

when $|x| < 1$. Using that fact, what function does the power series $\sum_{k=0}^{\infty} kax^{k-1}$ equal when it converges?

Puzzle/quote of the day

In honor of my daughter's birthday last week, here's a brainteaser:

When asked about his age, the mathematician Augustus de Morgan answered, "I was x years old in the year x^2." Given that de Morgan died in 1871, in what year was he born?

(It turns out that I too will turn x years old in the year x^2. What year was I born in?)