Merit Worksheet #27, 4/8/09

Parametric equations

1. From what you read for your preparation assignment, what does it mean for a curve to be defined by parametric equations?

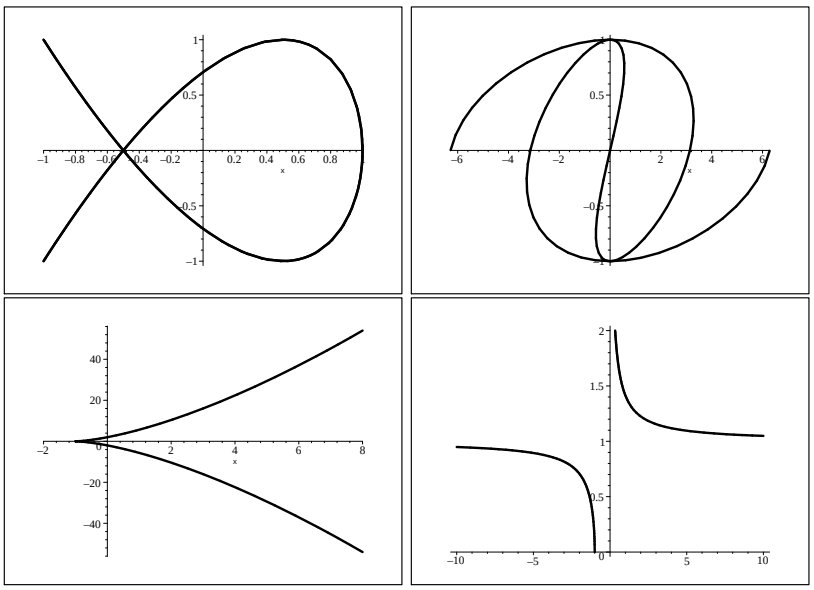
2. (a) Sketch the graph of the curve defined by $x = 2t - 1$, $y = t + 3$.
   (b) How would the graph be different if we added the restriction that $0 \leq t \leq 4$?
   (c) Can you write a single equation for the curve from part (a) that does not involve t at all?

3. (a) Sketch the graph of the curve defined by $x = \sin t$, $y = 3\cos t$.
   (b) Eliminate the parameter t from the pair of equations in part (a).
   (c) Given the answer to part (b), was your drawing in part (a) correct?

4. What advantages do you think that parametrically-defined curves have over curves defined by only a single equation involving x and y?

5. Match the following parametrically-defined curves to their graphs:

   (a) $x = t\cos t,\ y = \sin t,\ -2\pi \leq t \leq 2\pi$,
   (b) $x = 1/(t-1),\ y = \sqrt{(}t)$,
   (c) $x = \cos 2t,\ y = \sin 3t$,
   (d) $x = t^2 - 1,\ y = 2t^3$.



6. Find parametric equations describing the line segment from $(3, 1)$ to $(2, 4)$.

7. (IMPORTANT!) Suppose Objects 1 and 2 travel according to the parametrically defined curves given below:

$$\begin{cases} x &= t, \\ y &= t^2 - 1, \end{cases} \qquad \begin{cases} x &= 1 + s, \\ y &= 4 - s. \end{cases}$$

   (a) Do the paths the two objects trace out intersect each other in any place?

(b) Assuming that s and t both measure time, with the same units and starting at the same time, do the objects ever *collide*? If so, at what time?

(c) Answer the same questions if the paths are given by

$$\begin{cases} x &= t^2, \\ y &= 2t+1, \end{cases} \qquad \begin{cases} x &= s+2, \\ y &= 4s-3. \end{cases}$$

8. Find where the places, if any, where the curve

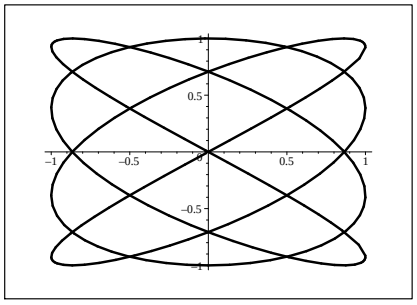
$$\begin{cases} x &= t^2-2t, \\ y &= t^3-3t^2+2t \end{cases}$$

intersects itself.

9. The graph below was traced out by the parametric equations

$$\begin{cases} x &= \sin 2t, \\ y &= \sin(3/2)t. \end{cases}$$

In which direction was the graph traced out? How long does t need to run before the graph starts repeating itself?



10. (a) Find a parametrization for (x, y) in terms of t for the curve $y^2 = x^3$ from $(4, 8)$ to $(1, 1)$.

(b) Find another parametrization for the same curve, but with the restriction that t runs only from 0 to 1.

Preparation for next time

On Friday we will begin Section 9.2 on Calculus and Parametric Equations. Please read the section up to, but not including, Theorem 2.2 on page 730. Prepare a reading question and answers to Problems 1, 9, and 15 for class.

Also, remember to work on your projects! Remember, if you turn something (anything–it doesn't have to be the whole project) in to me by Monday, then you will be guaranteed to receive your paper back with my comments in enough time to revise and resubmit it for the actual deadline of April 27. (And please consider doing one of the projects dealing with parametric curves. They're neat! Project #1 will also deal somewhat with parametric curves, and it will be posted as an option by this weekend.)

Quote of the day

"Mark all mathematical heads which be wholly and only bent on these sciences, how solitary they be themselves, how unfit to live with others, how unapt to serve the world." —Roger Ascham (1515–1568)