Merit Worksheet #30, 4/15/09

Arc length of parametric curves and surface area of solids of revolution

1. Suppose an object's speed is given by the function $v(t)$. How would you calculate the distance the object traveled between times $t = a$ and $t = b$? (This question comes from Calc I.)

2. What's the formula for the speed at which a parametric curve is traced out at time t? What's the formula for the length of a parametric curve between $t = a$ and $t = b$? What's the connection between the two answers?

3. Suppose $x = R\cos t$, $y = R\sin t$, $0 \le t \le 2\pi$.

   (a) Eliminate the parameter t to get an equation in x and y for the curve. What shape does the curve have?

   (b) Use the arc length formula to determine the length of the curve traced out.

4. Write out the formula for the area of the surface swept out when the curve $x = x(t)$, $y = y(t)$ is revolved around the x-axis (see the third equation from the bottom on page 738 of your text if you would like). Where does each part of that integral come from?

5. Using the formula you wrote down in the last problem, find the surface area of a sphere.

6. Find the length of the curve $y = x^2$ between $(0, 0)$ and $(1, 1)$.

7. Find the length of the curve $x = t^{3/2}$, $y = (1 - t)^{3/2}$, $0 \le t \le 1$.[1] Then find the area of the surface swept out when this curve is revolved around the x-axis.

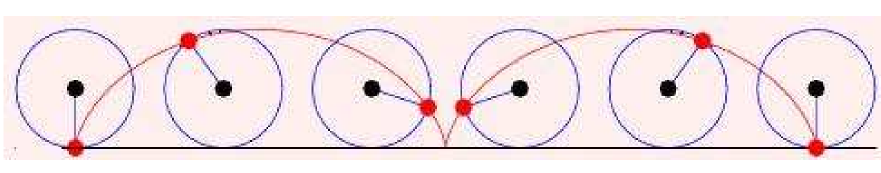
8. Find the length of the curve $x = t\cos t$, $y = t\sin t$, $-1 \le t \le 1$.

9. The ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
can be parametrized by the equations $x = a\cos t$ and $y = b\sin t$. Set up an integral representing the circumference of this ellipse, *but do not worry about evaluating it.*[2]

10. When you revolve an ellipse around one of its axes, you get what's called an *ellipsoid*. Using the same parametrization as in the last problem, find an integral representing the surface area of an ellipsoid (and don't worry about evaluating it).

11. Set up, but do not evaluate, an integral representing the length of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$.

12. The *cycloid* is the curve traced out by a point on the rim of a wheel as the wheel rolls along, as shown in this graphic from MathWorld:



[1] This is another parametrization of the *astroid*, the curve that appears on the last worksheet and in Project #1.

[2] Actually, the integral *can't* be evaluated exactly with what we've learned so far, but, in case you were wondering, the circumference can be approximated by $\pi\left[3(a + b) - \sqrt{(a + 3b)(3a + b)}\right]$.

Project #7 examines many properties of the cycloid, but we'll just look at one of them here. If the cycloid is given by the parametric equation $x = R(t - \sin t)$, $y = R(1 - \cos t)$, where R is the radius of the wheel, then how long is one arch of the cycloid?

13. Suppose we want to find the length of the curve $y = 1/x$ between $x = 1$ and $x = 2$.

    (a) Using the parameterization $x = t$, $y = 1/t$, $1 \leq t \leq 2$, write, but do not evaluate, an integral representing the arc length.

    (b) Starting with a power series for $\sqrt{1+x}$, approximate the integral you found in part (a).

Preparation for next time

Next time we will review for Monday's exam. There will be nothing to turn in, but please review all the material we've gone over from Sections 8.6–8.8 and 9.1–9.3 and bring your questions. Also, remember the Mock Exam tomorrow night from 6 to 8 PM in room 341 of Altgeld Hall.

Math songs of the day

I'm very good at integral and differential calculus,
I know the scientific names of beings animalculous;
In short, in matters vegetable, animal, and mineral,
I am the very model of a modern Major-General.
W. S. Gilbert (1836 - 1911), *The Pirates of Penzance*

You're an abacus
And my heart was counting on us.
Barenaked Ladies, "Adrift"

The teacher taught the alphabet,
We had to learn each letter,
The alphabet was not much fun,
But then things got much better:
The teacher said, "My children dear,
It's time for numbers now.
Can anybody count to ten?"
A small voice cried, "And how!"
"Just watch!" I cried,
"Before this class is done,
I will count all of the children one by one.
Yes! I'll count Gregory and Sue,
That makes number 1 and 2..."
The Count, "The Count's First Day at School," *Sesame Street*
(Available on YouTube. Check it out!)

Review Problems for the Final — Sections 6.2 and 6.3

In preparation for the final, worksheets from now until the final will usually contain problems to help you review. These problems will be typical of the types of problems that may appear on the final. At this point you should assume that you will not be allowed any aids during the test (note cards, cheat sheets, etc.), so it is VERY important that you review concepts we have learned throughout the semester, especially if you've been relying a lot on MathZone's hints as you do your homework. Because of time issues, and in an effort to motivate you to work with your classmates and not postpone thinking about these problems, I won't be posting solutions to these problems. If you get stuck in working a problem, let me or a fellow class member help you out.

The following problems are in random order and come from Sections 6.2 and 6.3. Good luck!

Evaluate the following integrals.

A. $\displaystyle\int \tan x \sec^3 x \, dx$ B. $\displaystyle\int e^x \sin 4x \, dx$ C. $\displaystyle\int_0^\pi \sin^4 x \, dx$

D. $\displaystyle\int \frac{2}{\sqrt{x^2-4}} \, dx$ E. $\displaystyle\int \sec x \, dx$ F. $\displaystyle\int \ln x \, dx$

G. $\displaystyle\int \tan x \, dx$ H. $\displaystyle\int x^2 \sqrt{x^2+9} \, dx$ I. $\displaystyle\int \sec^3 x \, dx$

J. $\displaystyle\int \frac{1}{\sqrt{1+4x^2}} \, dx$ K. $\displaystyle\int_0^1 x^2 e^{-3x} \, dx$ L. $\displaystyle\int \cos^3 x \sin^4 x \, dx$

M. $\displaystyle\int_0^2 \sqrt{4-x^2} \, dx$ N. $\displaystyle\int_1^2 x^2 \ln x \, dx$