Merit Worksheet #31, 4/17/09

Part I: A Group Review

I will assign your group a number. Based on your number, take 20 minutes to prepare a presentation to the class about the topics assigned to your group. Try to have as many people as possible involved in the presentation of your answers, and try to be both accurate and entertaining in your answers. To shake things up, ***this time you must include a jingle or slogan as part of your presentation***. You might also include things like the following (but only a jingle or slogan is required):

- drawings (diagrams, stick figures, comic strips, abstract art, etc.)
- skits
- limericks or other short poems
- dramatic readings of the text
- interpretive dance
- examples (real-life or dry as dust)
- references to current events, music, TV or movies
- etc.

Just keep it appropriate, and keep it correct. Let me know if you have any questions. Have fun!

Group 1: Power series Make a presentation, 10 minutes or less, teaching the class key life lessons about the following things.

- What a power series is.
- What an interval of convergence is, what it means, and how to compute it.
- How you can easily find power series for $f'(x)$ and $\int f(x)\,dx$ if you know a power series for $f(x)$.
- What happens to the radius of convergence when you take the derivative or integral of a power series.

Group 2: Taylor polynomials and error calculations Make a presentation, 10 minutes or less, teaching the class key life lessons about the following things.

- Functions whose power series you should memorize.
- What the nth-degree Taylor polynomial $P_n(x)$ is.
- How to compute the error/remainder term $R_n(x)$.
- How to estimate the error when using a Taylor polynomial to approximate a function.

Group 3: Applications of Taylor series, and parametric curves Make a presentation, 10 minutes or less, teaching the class key life lessons about the following things.

- How the graph of $f(x)$ compares to the graph of its nth-degree Taylor polynomial as n gets larger and larger.
- How to use power series to conjecture the value of an indeterminate-form limit like
$$\lim_{x\to 0}\frac{\sin(x^3)-x^3}{x^9}.$$
- How to use a power series to approximate a difficult definite integral like $\displaystyle\int_{-1}^{1}\frac{\sin x}{x}\,dx$.
- How to graph parametrically defined curves.

Group 4: Calc I and parametric curves Make a presentation, 10 minutes or less, teaching the class key life lessons about the following things.

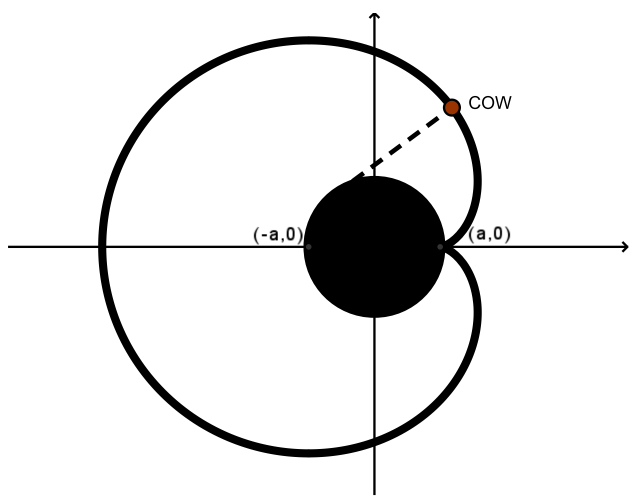
- How to eliminate the parameter in parametric curves.
- What $x'(t)$ and $y'(t)$ represent, and how to find the speed at which a parametric curve is traced out.
- How to find the slope dy/dx of a parametrically defined curve.
- How to find the second derivative d^2y/dx^2 of a parametrically defined curve (whatever you decided to do, please illustrate this with an example).

Group 5: Areas and arc length Make a presentation, 10 minutes or less, teaching the class key life lessons about the following things.

- How to find the area under a parametrically defined curve.
- What the formula is for the arc length of a parametric curve, and where it comes from.
- What the formula is for the area of the surface swept out when a parametric curve is revolved about the x-axis.
- What makes finding the surface area or arc length a usually difficult proposition.

Fun with cows and curves

Suppose that a rope of length πa joins a cow to the side of a circular water tank with radius a. For convenience let's suppose that the water tank is centered at the origin, and the rope is tied to the water tank at the point $(-a, 0)$. If the cow pulls the rope taut and walks around, its path resembles the curve shown below:



(Here the water tank is shown as the filled-in circle.) The curve is formed of three pieces: Part I is traced out according to the parametric equations

$$x = a(\cos t + t\sin t), \qquad y = a(\sin t - t\cos t)$$

as t varies between 0 and π; Part II is traced out according to these same equations, but as t varies between $-\pi$ and 0; finally, Part III is traced out according to the equations

$$x = a(\pi\cos t - 1), \qquad y = a\pi\sin t, \qquad \pi/2 \le t \le 3\pi/2.$$

The following problems will examine the details of such a setup.

1. a) Label the portions of the curve which belong to Parts I, II, and III.
   b) From the physical description of the cow's movement, what shape does Part III of the curve actually have?
2. If our cow is in the habit of pacing, how far will it travel as it starts from the point $(a, 0)$, walks at the end of its rope around the water tower, and again ends up at $(a, 0)$?
3. Suppose the farmer wants to enclose the cow's walking space within as small a rectangular pasture as possible, where the boundary fence is to have its sides parallel to the coordinate axes. Find the coordinates of the corners of the fence.
4. The cow can graze in the region inside its "walking curve" but outside the water tank. Find this area.
5. Now suppose the picture above is revolved around the x-axis.
   (a) Explain how the volumes and curves generated during the revolution relate to cows in space (in other words, what's physical significance of the solids and surfaces of revolution swept out?).
   (b) Find the area of the surface swept out by the cow's "walking curve."
   (c) Find the volume of the solid generated by the cow's grazing area.
6. Show that the equations given in the setup really do describe the motion of the cow as it walks around the water tank. How did we get them?

For next time

The exam is on Monday from 11 to 12:50 (our normal class time) in ***136 Burrill Hall***. Burrill Hall is located one block east of the Illini Union and is northwest of the Krannert Center, as shown on the map below:

Quote of the day

"And Lucy, dear child, mind your arithmetic. You know in the first sum of yours I ever saw there was a mistake. You had carried two, (as a cab is licensed to do,) and you ought, dear Lucy, to have carried but one. Is this a trifle? What would life be without arithmetic, but a scene of horrors? You are going to Boulogne, the city of debts, peopled by men who never understood arithmetic..."
—Writer and clergyman Sydney Smith (1771–1845), in an 1835 letter to a young friend.

Review Problems for the Final — Sections 6.4 and 6.6

These problems are provided in preparation for your final. They are typical of what you can expect from Sections 6.4 and 6.6. If you get stuck in working a problem, let me or a fellow class member help you out. Good luck!

Find the following antiderivatives.

A. $\displaystyle\int \frac{x^5+1}{x^4-x^2}\,dx$ B. $\displaystyle\int \frac{x^2+x-6}{x^2-1}\,dx$ C. $\displaystyle\int \frac{3x+8}{x^3+5x^2+6x}\,dx$

D. $\displaystyle\int \frac{4x-2}{16x^4-1}\,dx$ E. $\displaystyle\int \frac{2x+3}{x^2+2x+1}\,dx$ F. $\displaystyle\int \frac{2x^2+3}{x(x-1)^2}\,dx$

G. $\displaystyle\int \frac{5x^2-x+8}{x(x^2+4}\,dx$

Determine whether or not the improper integrals converge. Find the value of each that converges.

H. $\displaystyle\int_0^1 x^{-1/3}\,dx$ I. $\displaystyle\int_0^1 \frac{1}{x^{4/3}}\,dx$ J. $\displaystyle\int_1^\infty x^{-4/5}\,dx$

K. $\displaystyle\int_1^\infty \frac{1}{x^{6/5}}\,dx$ L. $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$ M. $\displaystyle\int_0^1 \ln x\,dx$

N. $\displaystyle\int_{-4}^4 \frac{2x}{x^2-1}\,dx$ O. $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2}\,dx$

Decide whether or not the integrals converge (remember the comparison test!).

P. $\displaystyle\int_1^\infty \frac{x}{1+x^3}\,dx$ Q. $\displaystyle\int_1^\infty \frac{x^2-2}{x^4+3}\,dx$ R. $\displaystyle\int_2^\infty \frac{x}{x^{3/2}-1}\,dx$

S. $\displaystyle\int_0^\infty \frac{\sin^2 x}{1+e^x}\,dx$ T. $\displaystyle\int_0^\infty e^{-x^2}\,dx$ U. $\displaystyle\int_0^\infty \frac{3}{x+e^x}\,dx$