Merit Worksheet #32, 4/22/09

Introduction to polar coordinates

Today's worksheet covers Section 9.4 (Polar Coordinates) in your text. Open up there (page 742), if you need to, and answer the following questions.

1. Plot the point given in polar coordinates, and find the corresponding cartesian coordinates for the point:

   (a) $\left(4, \frac{\pi}{2}\right)$ (b) $\left(-1, \frac{5\pi}{4}\right)$ (c) $\left(\frac{3}{2}, \frac{5\pi}{2}\right)$

2. In the problems below the rectangular coordinates of a point are given. Plot the point and find two sets of polar coordinates for the point (where your coordinates satisfy $0 \leq \theta < 2\pi$):

   (a) $(1, 1)$ (b) $(-6, 0)$ (c) $(5, 12)$

3. Express the given rectangular equations in polar form:

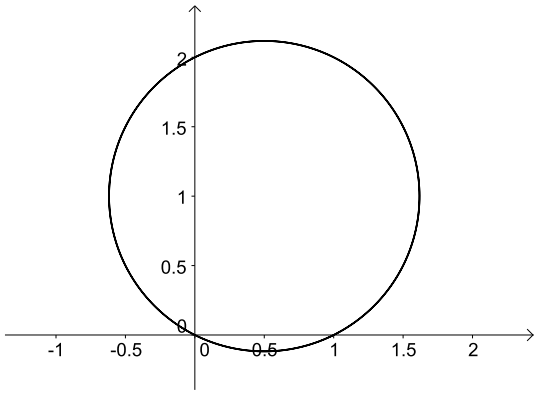
   (a) $x^2+y^2-4x = 0$ (b) $3x-y+2 = 0$ (c) $xy = 4$ (d) $(x^2+y^2)^2-9(x^2-y^2) = 0$

4. Express the given polar equations in rectangular form:

   (a) $r = 4$ (b) $r = 4\sin\theta$ (c) $\theta = \pi/6$ (d) $r^2 = \sin 2\theta$

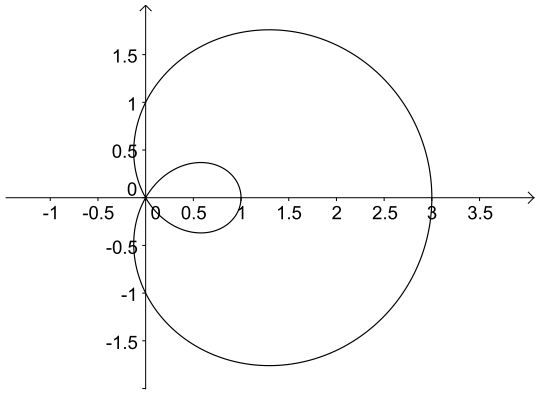
5. Write equations in both rectangular and polar forms for the line with x-intercept A and y-intercept B.

6. Which of the polar equations below describes the following graph?



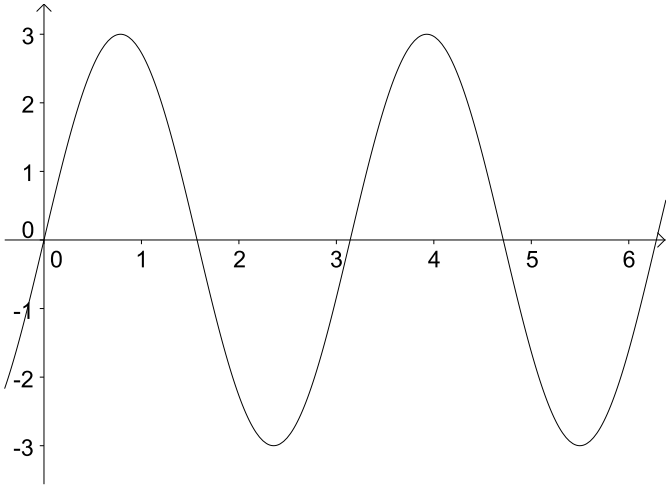
   (i) $r = 1$ (ii) $r = \frac{\sqrt{5}}{2}$ (iii) $r = 1 + \sin\theta$ (iv) $r = \cos\theta + 2\sin\theta$

7. Which of the polar equations below describes the following graph?



(i) $r = -1 + 2\cos\theta$ (ii) $r = 3 - 2\sin\theta$ (iii) $r = 2 + \cos\theta$ (iv) $r^2 = 9\cos\theta$

8. Shown here is a graph of $y = 3\sin 2x$:



Use the information contained in this graph to help you plot the polar curve $r = 3\sin 2\theta$.

9. (a) Plot the graph $y = 1 + \cos x$ between $x = 0$ and $x = 2\pi$ on a pair of x, y-axes.
   (b) In your drawing for part (a), erase the "x" label on the horizontal axis, and write θ in its place. Erase the "y" labelling the vertical axis, and write r in its place.
   (c) Using the graph you now have to aid you, graph the polar equation $r = 1 + \cos\theta$.

10. Find all points of intersection of the two polar curves:
    (a) $r = 1$, $r = \sin\theta$
    (b) $r = 1 + \cos\theta$, $r = 1 - \sin\theta$

11. Test the polar equation $r = 2\sin 5\theta$ for symmetry about the x- or y-axes, or about the origin.

Another very bad pun

Q: Whats a polar bear?
A: A rectangular bear after a coordinate transform.

For next time

Next time we will cover all of Section 9.5 (Calculus and Polar Coordinates). Please read the beginning of the section through Example 5.1. Then read the boxed area formula on page 758, together with Examples 5.3, 5.4, and 5.5. Then read the boxed arc length formula on page 762, together with Example 5.8. Prepare Exercises 1, 15, and 35 and a reading question to turn in.

Review Problems for the Final — Sections 8.4 and 8.5

These problems are provided in preparation for your final. They are typical of what you can expect from Sections 8.4 and 8.5. If you get stuck in working a problem, let me or a fellow class member help you out. Good luck!

Determine whether the following series converge absolutely, converge conditionally, or diverge.

A. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{3}{k!}$ B. $\displaystyle\sum_{k=0}^{\infty}(-1)^k 2^k$ C. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{2}{3^k}$

D. $\displaystyle\sum_{k=1}^{\infty}\frac{k}{k^2+1}$ E. $\displaystyle\sum_{k=0}^{\infty}\frac{3^k}{k!}$ F. $\displaystyle\sum_{k=3}^{\infty}(-1)^{k+1}\frac{4}{2k+1}$

G. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{4k+1}{5k-7}\right)^k$ H. $\displaystyle\sum_{k=1}^{\infty}\frac{e^{3k}}{k^{3k}}$ I. $\displaystyle\sum_{k=1}^{\infty}\frac{\cos k}{k^3}$

How big could the error be between the tenth partial sum and actual series value for the following?

J. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{4}{k^3}$ K. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{2}{k}$ L. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}e^{-k}$