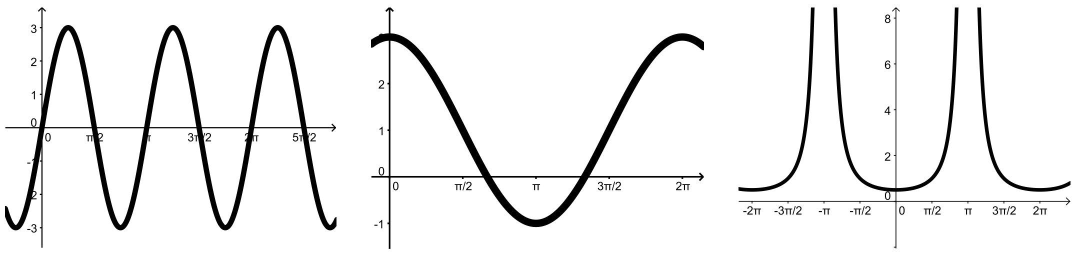
Merit Worksheet #33, 4/24/09

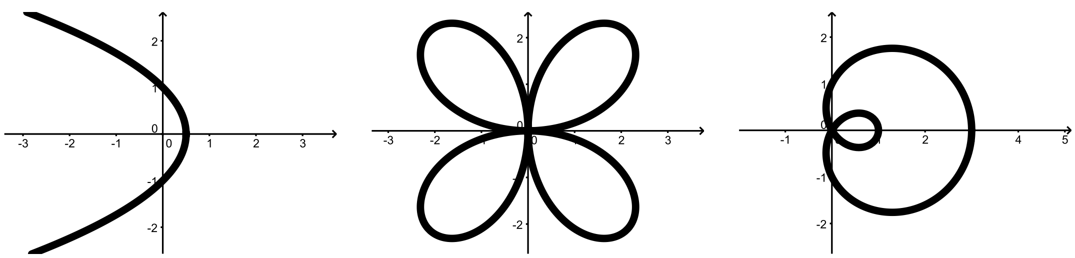
Calculus and polar curves

1. Shown below are the graphs of three curves in cartesian coordinates (i.e., $y = f(x)$), followed by the graphs of the same functions in polar coordinates (i.e., $r = f(\theta)$). Match the cartesian graph to the corresponding polar graph. What about each graph tells you you're right?

Cartesian curves $y = f(x)$



Polar curves $r = f(\theta)$



2. (a) Sketch the graph of $y = x/2$.
   (b) Using information from your graph in part (a), sketch the graph of $r = \theta/2$ on one of the polar grids at the end of the worksheet.

3. (a) Sketch the graph of $y = \sin x$.
   (b) Using information from your graph in part (a), sketch the graph of $r = \sin\theta$ on one of the polar grids at the end of the worksheet.

4. (a) Conceptual question: how would you find out where the points on a polar curve are that are the farthest away from the origin?
   (b) Two of the curves in Problem 1 are $r = 3\sin 2\theta$ and $r = 1 + 2\cos\theta$. Using the method you came up with in (a), find out where these curves are farthest away from the origin. Do the graphs agree?

5. (a) When you're given parametric equations $x(t)$ and $y(t)$, how do you compute dy/dx?
   (b) Now suppose you're given an equation $r = f(\theta)$. Can you come up with equations for $x(\theta)$ and $y(\theta)$?
   (c) Using your answers to parts (a) and (b), can you come up with a formula for dy/dx that works for polar curves?

6. Using what you know about slopes of polar curves, find where the tangent lines to the curves $r = 3\sin 2\theta$ and $r = 1 + 2\cos\theta$ are horizontal or vertical. Do the graphs accurately depict this?

7. (a) A check on your reading: how do you find the area enclosed by a polar curve?
   (b) The graph of $r = 3\sin 2\theta$ has some "petals." Find the area enclosed by one of the petals.
   (c) The graph of $r = 1 + 2\cos\theta$ has one loop enclosed in the other. Find the area between the two loops.
8. Find the area inside the graph of $r = 2\sin\theta$ but outside the graph of $r = 1$.
9. The curve whose equation in rectangular coordinates is

$$y^2 = x^2\left(\frac{a-x}{a+x}\right), \qquad a > 0,$$

   is called a strophoid.

   (a) Based on its equation in rectangular coordinates, how would you guess that this curve behaves near $x = -a$?
   (b) Show that the polar equation of this curve has the form $r = a\cos 2\theta\sec\theta$.
   (c) Come up with a rough sketch of this curve.
   (d) This curve has a loop. Find the area inside the loop when $a = 2$.
10. The curve whose equation in rectangular coordinates is

$$(x^2 + y^2)^2 = ax^2y, \qquad a > 0,$$

   is called a bifolium.

   (a) Show that the polar equation of this curve has the form $r = a\sin\theta\cos 2\theta$.
   (b) Come up with a rough sketch of this curve.
   (c) Find the area inside one of the curves loops when $a = 2$.

Preparation for next time

Next time we will continue looking at calculus and polar curves. Look over Section 9.5 again in your textbook. Though there is no exercise to turn in next time, please write down a reading question.

Joke of the day

An engineer, a physicist and a mathematician find themselves in an anecdote, indeed an anecdote quite similar to many that you have no doubt already heard. After some observations and rough calculations the engineer realizes the situation and starts laughing. A few minutes later the physicist understands too and chuckles to himself happily, as he now has enough experimental evidence to publish a paper. This leaves the mathematician somewhat perplexed, as he had observed right away that he was the subject of an anecdote, and deduced quite rapidly the presence of humor from similar anecdotes, but considers this anecdote to be too trivial a corollary to be significant, let alone funny.

Review problems for the final — Sections 8.6 and 8.7

A. Find the interval of convergence for each of the following series:

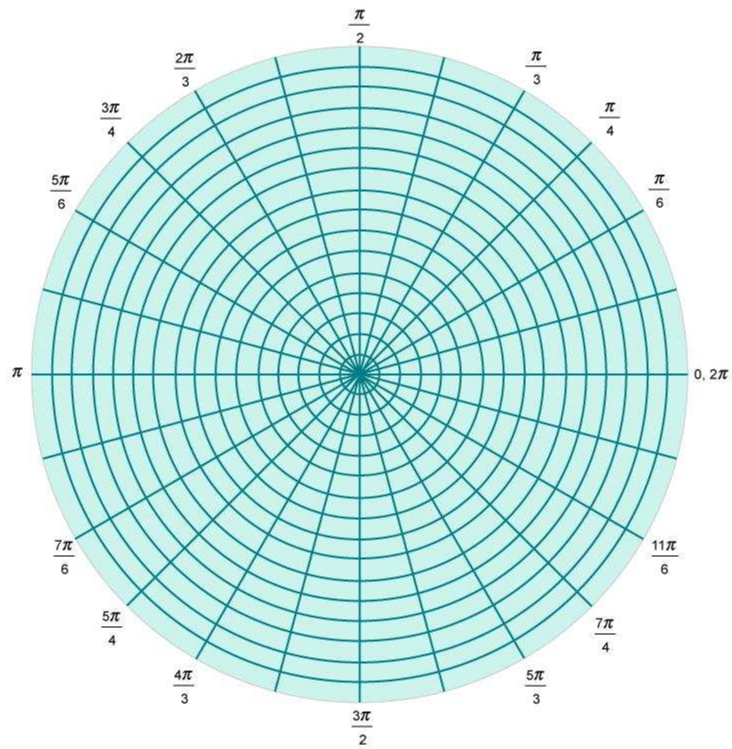
$$\text{(a)} \quad \sum_{k=0}^{\infty} \frac{2^k}{k!}(x-2)^k, \qquad \text{(b)} \quad \sum_{k=0}^{\infty} \frac{k}{4^k} x^k, \qquad \text{(c)} \quad \sum_{k=0}^{\infty} \frac{(-1)^k}{k3^k}(x-1)^k.$$

B. Find the 9th degree Taylor polynomial of $f(x) = \cos x$ about $c = \pi/2$. Also find the remainder term $R_9(x)$.

C. Find the Taylor series for $f(x)$ about the point $x = c$ in the following:

$$\text{(a)} \quad f(x) = \ln(1+x), \quad c = 0, \qquad \text{(b)} \quad f(x) = e^{2x}, \quad c = 1, \qquad \text{(c)} \quad f(x) = \frac{1}{(1-x)^2}, \quad c = 0,$$

$$\text{(d)} \quad f(x) = \frac{1}{1+x^2}, \quad c = 0, \qquad \text{(e)} \quad f(x) = \tan^{-1} x, \quad c = 0, \qquad \text{(f)} \quad f(x) = \frac{1}{x}, \quad c = 2.$$



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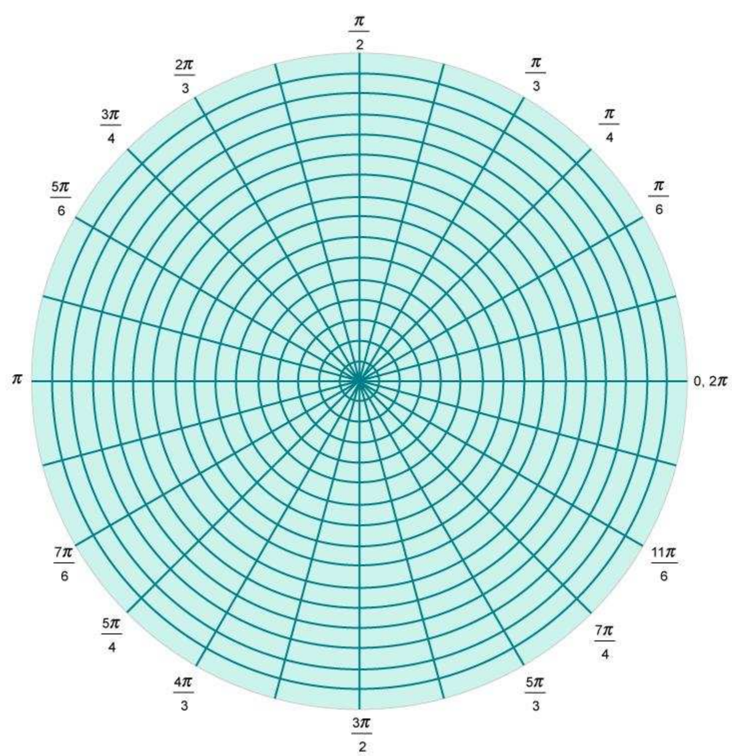

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