Merit Worksheet #35, 4/29/09

Conic sections

1. What exactly is a conic section? There are three (or four or five, depending on how you classify things) basic conic sections. What are they, and why are they called conic sections? (See page 764.) Make a sketch of each curve.

2. (a) Give a geometric definition of a parabola. (See page 764 in your text.)
   (b) Using only this geometric definition and the distance formula, find the equation of the parabola with directrix $y = -1$ and focus $(1, 1)$.
   (c) Now look at Theorem 7.1 on page 775 of your text. Explain how ellipses and hyperbolae follow a rule similar to the one parabolas do.
   (d) What happens to the shape of the the conic section

$$r = \frac{ed}{e\cos\theta + 1}$$

   as e approaches 0 from above? As e approaches ∞? (See Theorems 7.1 and 7.2 in your text.)

3. (a) Give a geometric definition of an ellipse. (See page 767 of your text.)
   (b) Using a piece of cardboard, two thumbtacks, a piece of string, and a pencil, how can you construct an ellipse?
   (c) Thinking about the setup from part (b), what happens to the shape of your ellipse if you place the thumbtacks far apart? What happens if you put them closer together? What happens if you put both thumbtacks at the same spot?

4. Summarize briefly the light/sound reflection properties of ellipses, parabolas, and hyperbolas, as described on pages 766–767 (parabolas), 769–770 (ellipses), and 772 (hyperbolas). Then visit the ellipse on the south side of the quad, in front of Foellinger Auditorium, and be amazed (or at least read about it in Wikipedia's article on "ellipse").

5. (a) Give a geometric definition of a hyperbola. (See page 770 in your text.)
   (b) Suppose the energy from an earthquake reaches one seismic observation station 20 seconds sooner than it reaches another (and we know how fast earthquake energy travels, which is roughly constant). How can we use this information to determine where the earthquake's epicenter is located? Why is this information *not* enough to determine the exact location of the epicenter? Would knowing how long it took the earthquake to reach a third station help?

6. (a) Find an equation of the form $r = f(\theta)$ that traces out the ellipse

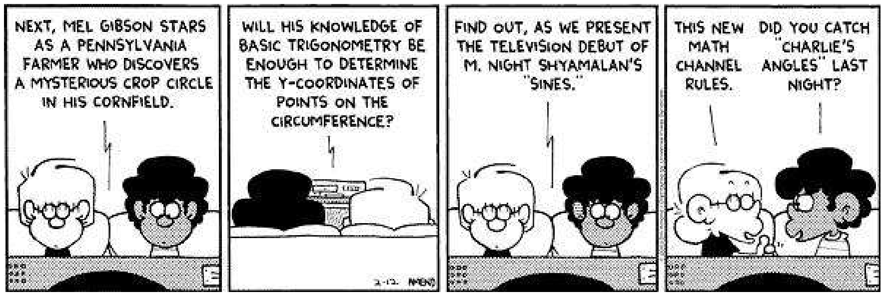
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

   (b) Show that the parametric equations $x = A\cos t$, $y = B\sin t$, $0 \le t \le 2\pi$ trace out an ellipse. (Hint: show that the curve satisfies the equation in part (a) if you choose a and b carefully.)
   (c) The parametric equations $x = A\cosh t$, $y = B\sinh t$, $-\infty < t < \infty$ trace out a portion of *which* conic section?[1]

7. Give a mathematical proof of the reflective property of parabolas.

[1] The answer to part (c), along with its similarities to part (b), explain in part why we call these the *hyperbolic trigonometric functions*.

Joke of the day



Preparation for next time

With this worksheet, we've covered the last of the material we'll cover during the semester. On Friday and on Monday and Wednesday of next week, we'll review for the final. In preparation for Friday's class, please look over everything you have (including the final review problems I've give you) for Chapter 6 (Techniques of Integration) and Section 7.1 (Differential Equations). Make a list of whatever questions you have to be answered, and identify some specific problems that either leave you stumped, or that you'd just like to see the solution to. There will be nothing to turn in.

Review problems for the final — Sections 9.2 and 9.3

A. Consider the curve given parametrically by $x = t^2 - 2$, $y = t^3 - t$.

   (a) Find the slope, dy/dx, of the tangent line to the curve when $t = 0$.

   (b) Find the slope at the point $(-2, 0)$.

   (c) Write a formula for the speed $v(t)$ at which the curve is traced out.

   (d) How fast is the curve being traced out when $t = 0$?

   (e) Is the curve concave up or concave down when $t = 0$?

   (f) Make a rough sketch of the curve when $-3 \le t \le 3$, and use arrows to indicate in which direction it is being traced out as t goes from -3 to 3.

B. Identify all points at which the curve $x = t^2 - 1$, $y = t^4 - 4t^2$ has a horizontal or vertical tangent. (Be sure to list both the x- and the y-coordinate of each point.)

C. Find the area between the curve $x = 3\sin t$, $y = 2\cos t$, $0 \le t \le \pi/2$ and the y-axis.

D. Write down, but do not evaluate, a definite integral representing the length of the curve traced out by $x = 2\cos t$, $y = 4\sin t$.

E. Find (i.e., evaluate) the length of the curve $x = 2 - t$, $y = t^2$ between $t = 0$ and $t = 2$.

F. Suppose the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is revolved around the x-axis. Write, but do not evaluate, a definite integral representing the surface area.

G. The curve $x = t^2 + 1$, $y = t$, $0 \le t \le 1$ is revolved around the x-axis. Find the area of the surface swept out.