

Merit Worksheet #36, 5/1/09

Review of Chapter 6 and Section 7.1

1. Find the following antiderivatives:

$$(a) \int \frac{1}{\sqrt{1-x^2}} dx \quad (b) \int \frac{1}{1+x^2} dx \quad (c) \int (e^{3x} - \cos(5x)) dx \quad (d) \int (\sec^2 x - \sec x \tan x) dx$$

2. Find the antiderivatives:

$$(a) \int x \sin 2x dx \quad (b) \int x^2 \ln x dx \quad (c) \int \tan^{-1} x dx$$

3. Find the antiderivatives:

$$(a) \int \cos^3 x dx \quad (b) \int \tan^5 x \sec x dx \quad (c) \int \tan x dx$$
$$(d) \int \sin x \sin 3x dx \quad (e) \int \cos^4 x dx \quad (f) \int \sec x dx$$

4. Find the antiderivatives:

$$(a) \int \frac{1}{9+x^2} dx \quad (b) \int \frac{\sqrt{x^2-4}}{x^2} dx \quad (c) \int \frac{1}{x^2\sqrt{1-4x^2}} dx$$

5. Find the antiderivatives:

$$(a) \int \frac{5x-2}{x^2-4} dx \quad (b) \int \frac{x^2-4x+1}{(x+1)(x-2)^2} dx \quad (c) \int \frac{x^5+6x^3-4x^2+x-2}{x^4+x^2} dx$$

6. Evaluate each of the following antiderivatives *in at least two separate ways*:

$$(a) \int x\sqrt{1-x} dx \quad (b) \int \frac{1}{x(\ln x)^3} dx \quad (\text{assume } x > 0) \quad (c) \int \frac{\ln x}{x} dx$$
$$(d) \int \sin x \cos x dx \quad (e) \int \frac{x}{9+x^2} dx$$

7. (a) What makes an integral improper? Give two examples of different ways an integral can be improper.

(b) Determine whether or not the improper integrals converge. If they converge, find their value:

$$(i) \int_0^{\infty} e^{-x} dx \quad (ii) \int_0^1 \frac{1}{x^{3/2}} dx \quad (iii) \int_0^2 \frac{x}{x^2-1} dx$$

8. What was that rule again? The integral $\int_1^{\infty} \frac{1}{x^p} dx$ converges if p is...

9. Do the following integrals converge or diverge?

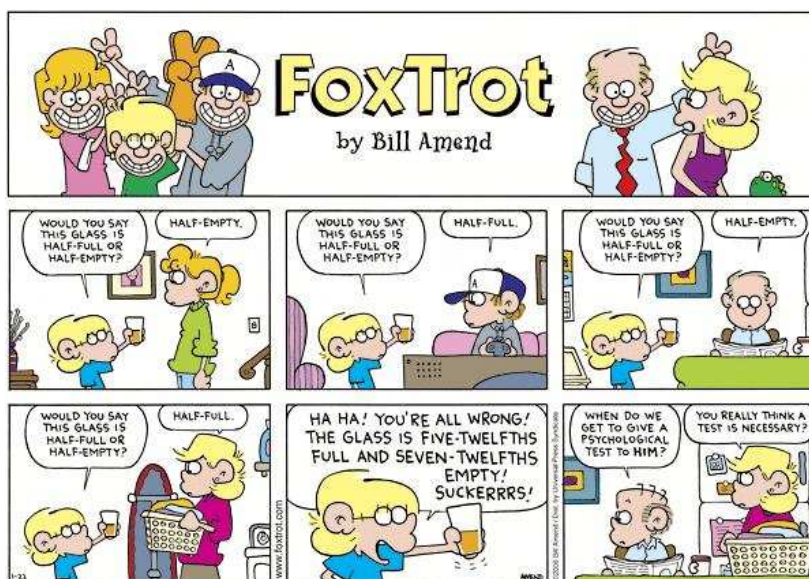
$$(a) \int_{37}^{\infty} \frac{\sqrt{x}}{x-1} dx \quad (b) \int_2^{\infty} e^{-x^3} dx \quad (c) \int_{-\infty}^{\infty} \frac{|\sin 4x|}{1+x^2} dx \quad (d) \int_0^{\infty} \frac{1}{x^2+x+1} dx$$

10. (a) What is a differential equation? What does the solution to a differential equation look like?
 (b) True or false? $y = x^3 - 3x^2 + x + 4$ is a solution to the differential equation $y'' + y' + y = x^3 + x - 1$.
 (c) True or false? $x = 3$ is a solution to the differential equation $(y')^2 - 3y' = 0$.
 (d) Solve the following initial value problem:

$$y'(x) = 4y(x), \quad y(0) = 10.$$

11. Radioactive decay also obeys the differential equation $y' = -ky$. At 7 AM 100 grams of radioactive material are placed under observation. By 10 AM the amount has decayed to 90 grams.
- (a) Find an equation for the amount of radioactive material t hours after 7 AM.
 (b) What is the half-life of the substance, i.e., how long does it take for the half of the original material to decay?

Joke of the day



Preparation for next time

On Monday we'll review material for the final from Chapter 8. Please look over everything you have (including the final review problems I've give you) for Chapter 8 (Infinite Series). Make a list of whatever questions you have to be answered, and identify some specific problems that either leave you stumped, or that you'd just like to see the solution to. There will be nothing to turn in.