Merit Worksheet #38, 5/6/09

Review of Chapter 9

1. (a) Sketch the graph of the curve defined by $x = 2t - 1$, $y = t^2 + 3$, $0 \leq t \leq 4$.
   (c) Eliminate the parameter to get an equation just in terms of x and y.

2. Say a curve is defined by $x = \sin^3 t$, $y = 3\cos t - 2$. Eliminate the parameter to get an equation just in terms of x and y.

3. Say a curve is traced out by a pencil whose position is given by

$$x = t^3, \qquad y = 1 - t^2, \qquad -\infty < t < \infty.$$

   (a) Find an equation for the line tangent to this curve at the point $(8, -3)$.
   (b) Find out if this curve has any horizontal or vertical tangents.
   (c) Find out the velocity (horizontal and vertical components) **and** speed at of the pencil at $t = 2$.
   (d) Find d^2y/dx^2 for this curve. Is the curve concave up or concave down at $(8, -3)$?

4. Suppose $x = t^2 - 2t$, $y = t^3 - 12t$.

   (a) Find the area under the curve between $t = -2$ and $t = 0$.
   (b) Find the area *above* the curve between $t = 2$ and $t = 3$.
   (c) Write, but don't worry about evaluating, an integral for the length of this curve between $t = 0$ and $t = 2$.

5. Consider the curve traced out by $x = \sin^3 t$, $y = \cos^3 t$.

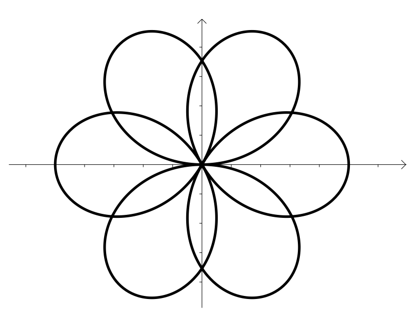
   (a) What's the second derivative of the curve when $t = \pi/4$?
   (b) Find the area under the curve between $t = 0$ and $t = \pi/2$.
   (c) Find (i.e., evaluate) the length of this curve between $t = 0$ and $t = \pi/2$.

6. Convert the polar equation into a cartesian one, and the cartesian equation into a polar one:

$$3x^2 - y + 2 = 0, \qquad\qquad r = 4\sin\theta.$$

7. Make a rough sketch of the polar curve $r = 1 + \cos\theta$.

8. Shown below is the graph of $r = \cos\left(\frac{3}{2}\theta\right)$.



   (a) Find the slope dy/dx when $\theta = \pi/2$.

(b) Write, but don't evaluate, an integral for the area inside one of the six loops.

9. (a) Give a geometric definition of a parabola.

   (b) Using only this geometric definition and the distance formula, find the equation of the parabola with directrix $x = 1$ and focus $(5, 0)$.

10. Explain what parabolas, ellipses, and hyperbolas all have to do with a focus and a directrix. Explain what eccentricity is. What values of eccentricity correspond to parabolas? ellipses? hyperbolas?

11. Find the equation of the conic section with directrix $y = 0$, focus $(0, 2)$, and eccentricity 3.

Notes on the final

The final will be held in ***138 Henry Administration Building***, on Tuesday, May 12, from 7 to 10 PM. It will be roughly twice the length of a midterm exam and will cover the entire semester pretty uniformly, though there may be one more question on polar coordinates/conic sections than there otherwise would have been. I will have my usual office hours tomorrow from 4 to 5 PM, and you're always welcome to email me with questions. Good luck studying!