

Comments on written homework assignment #3

The problem was this:

Use the definition of the derivative (a.k.a., slope predictor function) to find the derivative of $f(x) = x^3$ at every x . Do not just invoke the power rule; show all your work.

Here's one possible solution:

Because we were told to use the definition of the derivative, we start by setting up the limit of the difference quotient:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}. \end{aligned}$$

Now all we need to do is find this limit by simplifying the expression and using the limit rules:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 + 3x \cdot 0 + 0^2 \\ &= 3x^2. \end{aligned}$$

Note that in taking the limit we were able to just plug 0 in once the fraction was simplified because what we had was a polynomial, and polynomials are continuous.

Some comments

1. What would the world be like if no one used verbs? Does the following make sense: "I ... you ... well on your test; I ... fun ... them"? In math, the verbs are the 'equals' signs ('='). Notice how many I used in the solution? Many of you wrote out everything I did above, except for the equals signs. I didn't take off any points for that, but it really is just like speaking without verbs; I may be able to understand what you're saying, but it'd just be a lot better if you put them in.

2. Just as on the last homework, let me point out that I wrote 'lim_{h→0}' in front of every step until I had actually used the limit laws to plug 0 in. You need to write the 'lim' at each step. It's not just enough to write it at the beginning and/or the end; you need to write it at every step in which it belongs (which is pretty much *every* step).

3. Some people, on the other hand, will get to the end and say something like

$$\lim_{h \rightarrow 0} = 3x^2.$$

The limit of *what*? It doesn't make sense to write a limit without writing out what it is the limit of.