Math 220 DD6 Practice Exam #4

Nov. 15, 2006

Work each of the following problems just like you would if this were your real exam on Friday. No scientific calculators, notes, books, or similar helps. You will have 50 minutes in which to complete the test.

1. Let f be a continuous function on a closed interval $[a, b]$. Recall that if we split $[a, b]$ into subintervals, where each one has length at most Δx, then M_i represents the maximum value of $f(x)$ on the ith subinterval, and m_i represents the minimum value of $f(x)$ on the ith subinterval. Also recall that $E_(\Delta x)$ is the maximum change function, defined by $E_f(\Delta x) = \max_i(M_i - m_i)$.

Your question: What is the basic fact about $E_f(\Delta x)$ that is needed for the integral $\int_a^b f(x)\, dx$ to exist?

2. Evaluate $\int_2^5 e^{-t^2}\, dt + \int_5^2 e^{-x^2}\, dx$.

3. Evaluate the definite integrals:

a) $\int_2^3 2x\ dx$

b) $\int_0^{\pi/2} \sin x\ dx$

4. Let $f(x) = x^3$. Find the average value of f on the interval $[0, 2]$ and the point where f takes that average value.

5. a) Write a formula for a function of x on the real line for which the derivative is e^{-x^2} and for which the function's value is 0 when $x = 0$.

b) Find the derivative

$$\frac{d}{dx}\int_0^{\sin x} \sqrt{1+t^2}\, dt$$

6.

a) Change the limits of integration and evaluate $\int_0^1 \cos(\pi x^2)(2\pi x)\, dx$**.**

b) Solve $\int_2^3 x^2\sqrt{1+x^3}\, dx$**.**

c) Find the antiderivative: $\int x(x+3)^7\, dx$

7. Find the area of the region between the curves $y = x^2$ and $y = 2x + 8$.

8. a) Find the value of the integral $\int_1^2 \sqrt{x}\, dx$.

b) For $\Delta x = 1/5$, write out the Riemann sum (evaluating at the left) for $\int_1^2 \sqrt{x}\, dx$. Do not simplify.

c) For $\Delta x = 1/5$, write out the trapezoidal sum using the method discussed in class. Do not simplify.

9. Suppose the mass density of a 3-meter rod with the left end at $x = 0$ is given by $\rho(x) = x^2$ kg/m. Find the mass of the rod.

10. Find the volume of the solid you get by rotating the area between the line $y = x$ and $y = x^2$ about the y-axis.

Selected answers

1. $\lim_{\Delta x \to 0} E_f(\Delta x) = 0$
2. 0
3. a) 5
   b) 1
4. $\overline{y} = 2$, which happens at $x = \sqrt[3]{2}$
5. a) $\int_0^x e^{-t^2}\ dx$
   b) $\sqrt{1 + \sin^2 x} \cos x$
6. a) 0
   b) $\frac{2}{9}(28^{3/2} - 9^{3/2})$
   c) $\frac{1}{9}(x+3)^9 - \frac{3}{8}(x+3)^8 + C$
7. 36
8. a) $\frac{2}{3} \cdot 2^{3/2} - \frac{2}{3}$
   b) $\frac{1}{5}(\sqrt{1} + \sqrt{1.2} + \sqrt{1.4} + \sqrt{1.6} + \sqrt{1.8})$
   c) $\frac{1}{5}\left(\frac{1}{2}\sqrt{1} + \sqrt{1.2} + \sqrt{1.4} + \sqrt{1.6} + \sqrt{1.8} + \frac{1}{2}\sqrt{2}\right)$
9. 9 kg
10. $\pi/6$