Merit Worksheet #7, 9/11/06

LIMITS

1. For each of the following limits, answer the following:
   (i) What is the limit of just the numerator of the fraction?
   (ii) What is the limit of the denominator of the fraction?
   (iii) What is the limit of the entire fraction?
   (iv) Can you answer part (iii) just by looking at your answers in parts (i) and (ii)?

   a) $\lim_{h\to 0} \frac{\sin h}{h}$

   b) $\lim_{x\to 5} \frac{x^2-25}{x-5}$?

   c) $\lim_{t\to 0} \frac{1-\cos t}{t}$

   d) $\lim_{h\to 0} \frac{\sqrt{3+h}-\sqrt{3}}{h}$

   Be **SURE** you understand the algebra trick you need to solve the limit in part (d) above. It's a classic homework/test question!

2. Find the following limits:

   a) $\lim_{\theta\to 0} \frac{\sin(3\theta^2+\theta)}{3\theta^2+\theta}$

   b) $\lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta^2}$

   c)
   $$\lim_{\theta\to 0} \frac{\sin(5\theta)\sin(3\theta^2)}{\theta^2\cos(2\theta)\sin(2\theta)}.$$

THE INTERMEDIATE VALUE THEOREM

3. Suppose $q(x)$ is a rational function, with $q(-1) = 1$ and $q(1) = -1$. Can you say anything about if and/or where $q(x)$ equals zero?

4. By plugging in some test values of x and using the Intermediate Value Theorem, give an interval or intervals in which $2x^3 - 10x + 7$ has a root.

5. Bugs Meany is excited, because he knows he's got Encyclopedia Brown stumped this time. "Hey," he says to the sleuth in class one day at the U of I(daville), "I'll bet you can't solve the equation

   $$\frac{x}{2\pi} = \sin x;$$

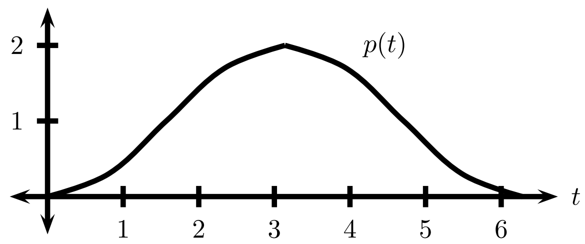
   I'll bet you can't even tell me if it *has* a solution."

   "Hmm," says Encyclopedia, leaning back in his chair and briefly closing his eyes. After a moment he opens them, and to Bugs' dismay, he answers, "Well, Bugs, you're right—other than the obvious solution of $x = 0$, I can't solve that equation by hand. But I can tell you that it does have at least two solutions, one between $-\pi$ and $-\pi/2$, and one between $\pi/2$ and π."

   HOW DID ENCYCLOPEDIA BROWN KNOW?

THE DERIVATIVE AND RATES OF CHANGE

6. Below is the graph of $p(t)$, where t measures time in hours, and $p(t)$ is the position of the newly-discovered Illinois trigonometric garden slug, measured in feet.



Answer the following questions:

a) Which of the following denotes or calculates an **average velocity** of the slug?
(i) $\frac{p(4.71)-p(2.36)}{4.71-2.36}$ (ii) $p'(3)$ (iii) $p(6.28)$

b) Which of the following denotes or calculates an **instantaneous velocity** of the slug?
(i) $\frac{p(4.71)-p(2.36)}{4.71-2.36}$ (ii) $p'(3)$ (iii) $p(6.28)$

c) At which time(s) does the slug reach its top speed? What would you estimate that speed to be?

d) On approximately which time interval(s) is the slug speeding up? On approximately which interval(s) is it slowing down? How can you tell?

7. Suppose $s(t)$ is the position of a particle on the x-axis at time t, where $s(t) = 3t^2 - 4t + 1$, and $0 \le t \le 5$. Answer the following:

a) At what position is the particle when $t = 5$?

b) When, if ever, does the particle pass through $x = 0$?

c) Find a formula for the speed of the particle, as a function of t.

d) What is the speed of the particle when $t = 2$?

e) Does the particle ever stop? If so, when?

f) Find a formula for the acceleration of the particle.

g) When, if ever, is the particle accelerating? When, if ever, is the particle decelerating?

DERIVATIVE STUFF

8. Which of the following are ways of writing the derivative of $f(x)$?
(a) $f'(x)$ (b) $Df(x)$ (c) $\frac{df}{dx}$

9. Using the product and sum rules of derivatives (and **NOT** the power rule, in case you already know it), find

$$\frac{d}{dx}(x^2 + 3x - 7)(3x^2 - 6).$$

10. Using the product rule (and **NOT** the power rule), find the derivatives of (a) $f(x) = x^3$, and (b) $f(x) = x^4$.

Remember, you have an exam next Friday during lecture. I'd suggest you study your homework, the sections in your text covering the lecture, the actual lecture notes, and your Merit worksheets (all of the worksheets, along with selected answers, can be found at http://www.math.uiuc.edu/∼mbarrus2/teaching.html). Our worksheet next time will be a practice exam, which you'll take during the first hour and we'll discuss during the second hour. Be sure to study!!!

Quote for the day: "An expert is someone who knows some of the worst mistakes that can be made in his subject, and how to avoid them." *Physics and Beyond*, Werner Heisenberg (1901-1976).

Selected answers

1. a) (i) 0, (ii) 0, (iii) 1, (iv) no
   b) (i) 0, (ii) 0, (iii) 10, (iv) no
   c) (i) 0, (ii) 0, (iii) 0, (iv) no
   d) (i) 0, (ii) 0, (iii) $\frac{1}{2\sqrt{3}}$ (iv) no

2. a) 1
   b) 1
   c) 15/2

3. No; it may never cross the x-axis (consider $q(x) = 1/x$, for example. Note that the intermediate value property does **not** apply here, for rational functions aren't always continuous.

4. Solutions lie in the intervals $[-3, -2]$, $[0, 1]$, and $[1, 2]$.

5. Encyclopedia Brown looked at the function $f(x) = \sin x - x/2\pi$. He saw that $f(-\pi) = 1/2$, $f(-\pi/2) = -3/4$, $f(\pi/2) = 3/4$, and $f(\pi) = -1/2$, and so by the intermediate value property he knew that $f(x)$ has to equal 0 somewhere between $-\pi$ and $-\pi/2$, and somewhere between $\pi/2$ and π. Now if $f(x) = 0$, then $\sin x - x/2\pi = 0$, which means that $\sin x = x/2\pi$, so the equation has a solution.

6. a) (i)
   b) (ii)
   c) The top speeds happen where the slope is the steepest; this happens at about $t = 1.5$ hours and $t = 4.5$ hours, where the speeds are each about 1 foot/hour.
   d) Speeding up on about $[0, 1.5]$ and on about $[3.1, 4.5]$, because the slopes are getting steeper; slowing down on about $[1.5, 3.1]$ and $[4.5, 6.2]$, since the slopes are getting closer to 0.

7. a) $s(5) = 56$
   b) At $t = 1/3$ and $t = 1$
   c) The velocity is given by $s'(t) = 6t - 4$, so the speed is $|s'(t)| = |6t - 4|$.
   d) When $t = 2$, the speed is 8.
   e) The speed will be 0 when $t = 2/3$.
   f) The acceleration is given by $s''(t) = 6$.
   g) The particle is always accelerating.

8. All three are ways of writing the derivative.

9. $(2x + 3)(3x^2 - 6) + (x^2 + 3x - 7)(6x)$

10. Since $x^3 = x \cdot x^2$, the product rule says

$$\frac{d}{dx}x^3 = 1 \cdot x^2 + x \cdot 2x = 3x^2.$$

Since $x^4 = x^2 \cdot x^2$, the product rule says

$$\frac{d}{dx}x^4 = 2x \cdot x^2 + x^2 \cdot 2x = 4x^3.$$