Merit Worksheet #29, 11/8/06

A whole bunch of calculus problems...

1. a) Suppose p and h are constants. What is the volume of the the solid generated by rotating around the x-axis the region under the line $y = \sqrt{2ph}$ between $x = 0$ and $x = h$. What shape is swept out?

   b) What is the volume of the solid generated by rotating around the x-axis the region under the parabola $y^2 = 2px$ between $x = 0$ and $x = h$?

   c) What percentage of your answer to part (a) is your answer to part (b)? Looking at figure 6.2.35 on page 417 of your text, does this seem reasonable?

2. Let's suppose that you're in the business of making wedding bands for mathematicians. In order to make the bands, you bore a hole of radius 1.5 cm out of a solid sphere (of gold or titanium, naturally) having radius 2.5 cm. What's the volume of such a ring?

3. What's the volume of the solid generated by rotating around the line $x = 3$ the region bounded by the curves $x = y^2$ and $y = x^2$?

4. What's the volume of the solid generated by rotating around the line $y = -1$ the region bounded by the curves $y = x^2$ and $y = 8 - x^2$?

5. Find the work done by exerting a force of $\sin \pi x$ on a particle along the x-axis, from $x = -1$ to $x = 1$.

6. A spring has a natural length of 2 ft, and a force of 15 lb is required to hold it compressed at a length of 18 in.

   a) What is the equation for the force that must be exerted on the spring to hold its right end x units past its natural position?

   b) How much work is done in stretching the spring from its natural length to a length of 3 ft?

7. Suppose the force necessary to hold a 1000-lb weight a distance r above the earth's center is given by $F(r) = 16 \times 10^9/r^2$. What is the amount of work (in foot-pounds) done in lifting this weight from an orbit 1000 mi above the earth's center to one 2000 mi above the earth's center?

8. A conical tank is resting on its base, which is at ground level, and its axis is vertical. The tank has radius 5 ft and height 10 ft. Compute the work done in filling this tank with water, which has density $\rho = 62.4$ lb/ft^3, pumped in from found level.

9. What's the work necessary if the water from the tank (which was completely filled in Problem 8) is now to be pumped out the top of the cone?

10. A tank whose lowest point is 10 ft above the ground has the shape of a cup obtained by rotating the parabola $x^2 = 5y$, $-5 \le y \le 5$, around the y-axis. The units on the coordinate axes are in feet. How much work is done in filling this tank with oil of density 50 lb/ft^3 if the oil is pumped in from ground level?

Quote of the day: "All great theorems were discovered after midnight." —Adrian Mathesis

Selected answers

1. a) $V = \pi \int_0^h \sqrt{2ph}^2 \, dx = 2\pi ph^2$; a cylinder is swept out.

   b) $V = \pi \int_0^h \sqrt{2px}^2 \, dx = \pi ph^2$

   c) 50%

2. $V = \pi \int_{-2}^{2} (\sqrt{2.5^2 - x^2}^2 - 1.5^2) \, dx = 32\pi/3$

3. $V = \pi \int_0^1 [(3 - y^2)^2 - (3 - \sqrt{y})^2] \, dy = 17\pi/10$

4. $V = \pi \int_{-2}^{2} ([8 - x^2 - (-1)]^2 - [x^2 - (-1)]^2) \, dx = 640\pi/3$

5. $W = \int_{-1}^{1} \sin \pi x \, dx = 0$

6. a) $F(x) = 30x$

   b) $W = \int_0^1 30x \, dx = 15$ ft-lbs

7-10. For solutions to work story problems, see the next worksheet.