

Canonical decomposition of graphs and antimagic labeling

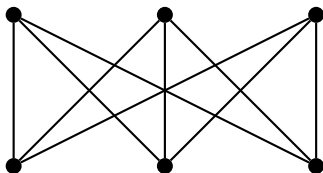
Michael D. Barrus

Department of Mathematics
University of Illinois at Urbana–Champaign

2008 AMS Fall Central Section Meeting
October 18, 2008

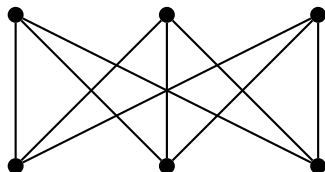
Magic labelings

2	7	6
9	5	1
4	3	8



Magic labelings

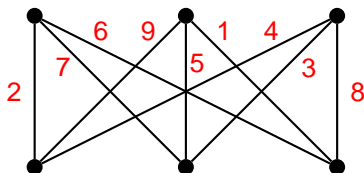
2	7	6	↗ 15
9	5	1	→ 15
4	3	8	→ 15
↓ 15	↓ 15	↓ 15	↘ 15



In a **magic square**, the sums along each row, column, and diagonal are the same.

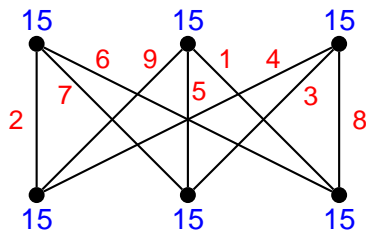
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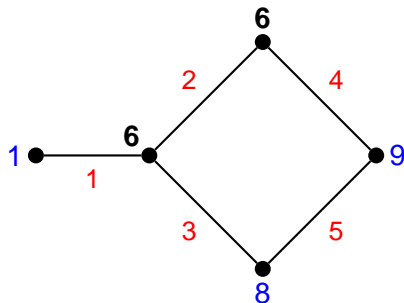
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In a **magic labeling** of a graph G , the edges are labeled with $1, \dots, |E(G)|$ so that all vertex sums are the same.

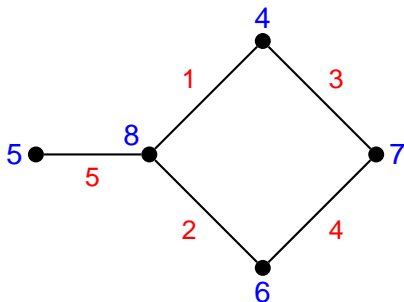
If such a labeling exists, then G is **magic**.

Antimagic labelings



In an **antimagic labeling** of a graph G , the edges are labeled with $1, \dots, |E(G)|$ so that all vertex sums are distinct.

Antimagic labelings



In an **antimagic labeling** of a graph G , the edges are labeled with $1, \dots, |E(G)|$ so that all vertex sums are distinct.

If such a labeling exists, then G is **antimagic**.

The conjecture

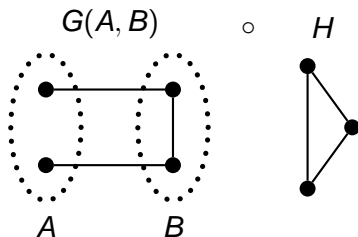
Conjecture (Hartsfield, Ringel, 1990)

Every connected graph other than K_2 ($o—o$) is antimagic.

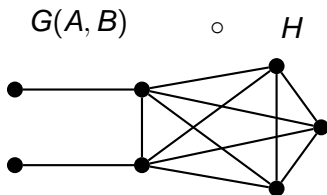
Proved for

- Cycles, paths, stars, complete graphs [Hartsfield, Ringel, 1990]
- Graphs with large minimum or maximum degree [Alon et al., 2004]
- Toroidal grids [Wang, 2005]
- Graphs with a K_3 -factor [Hefetz, 2005]
- Lattice grids and prisms [Cheng, 2007]
- Special graph products [Wang, Hsiao, 2008]
- Regular bipartite graphs [Cranston, 2008]

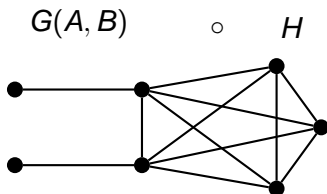
The canonical decomposition



The canonical decomposition



The canonical decomposition



Theorem (Tyshkevich, 1980)

Every graph can be written uniquely as a composition

$$G_1(A_1, B_1) \circ \cdots \circ G_k(A_k, B_k) \circ H$$

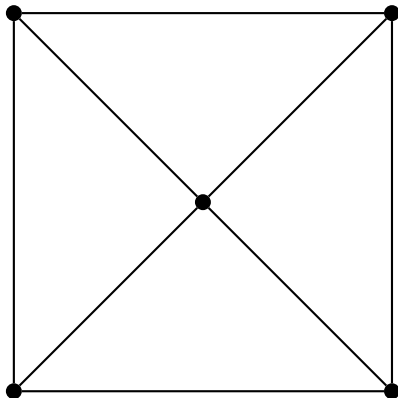
of indecomposable graphs.

A dominating vertex

Theorem (Alon et al., 2004)

If $G (\neq K_2)$ has a vertex which is adjacent to all other vertices, then G is antimagic.

Pf:

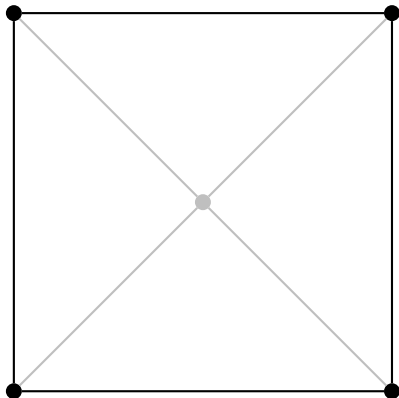


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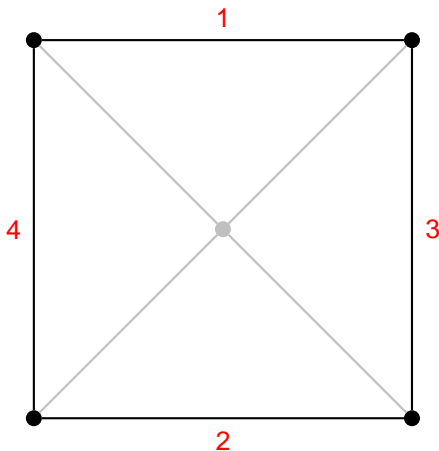


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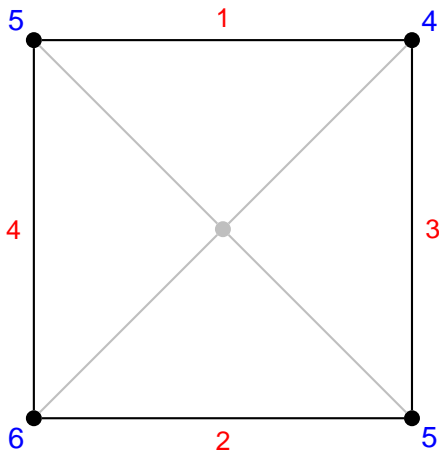


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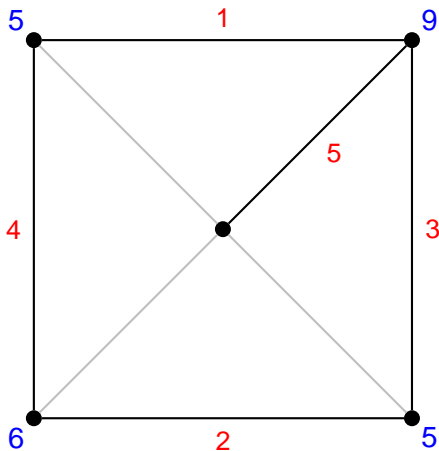


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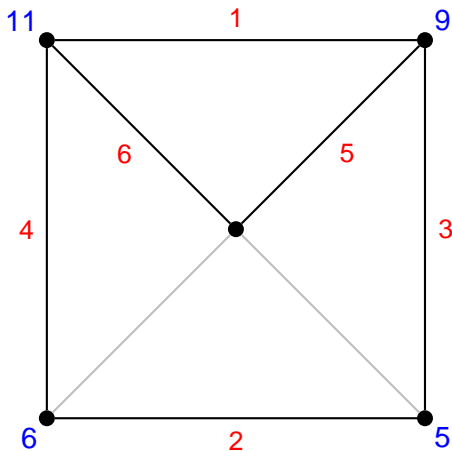


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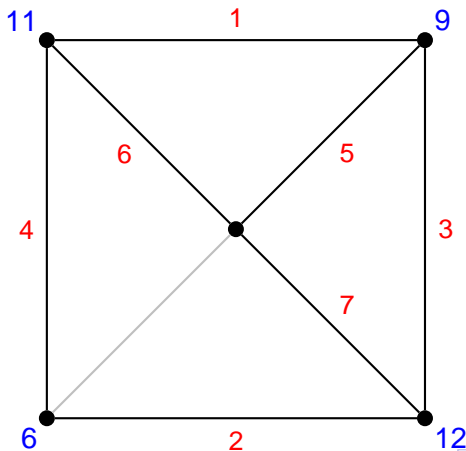


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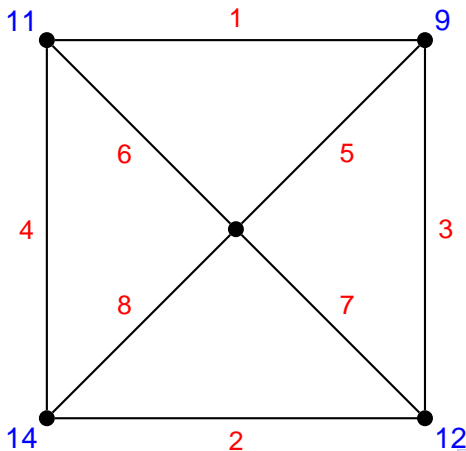


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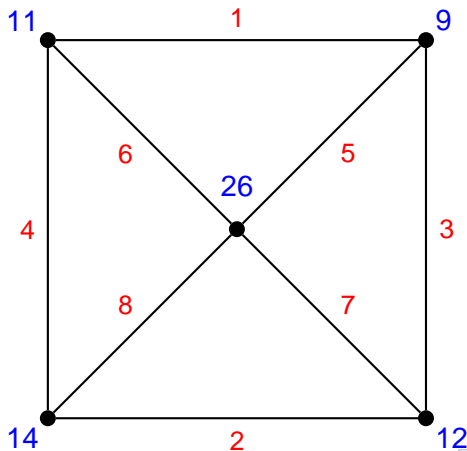


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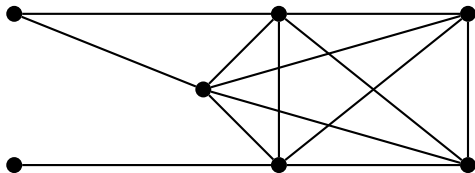
Pf:



The result

Theorem

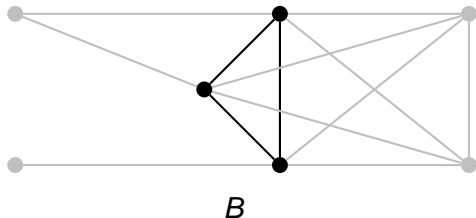
If connected G ($\not\cong K_2$) has a clique B such that every vertex in G is adjacent either to all vertices in B or to no vertex outside B , then G is antimagic.



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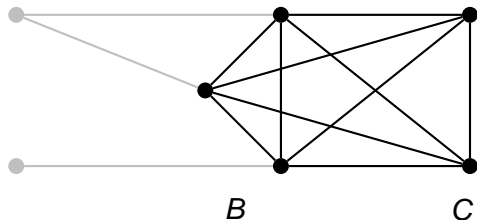


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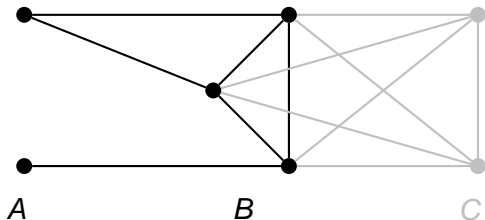


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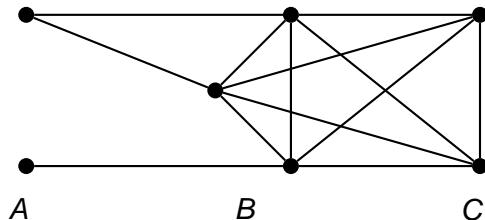


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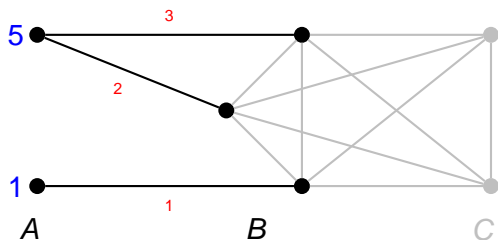


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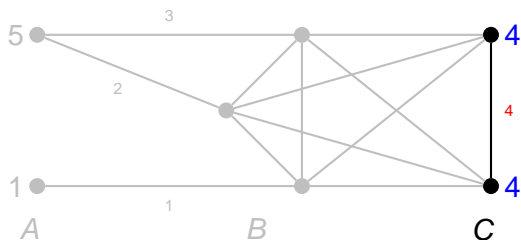


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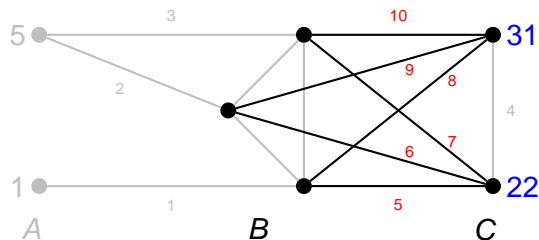


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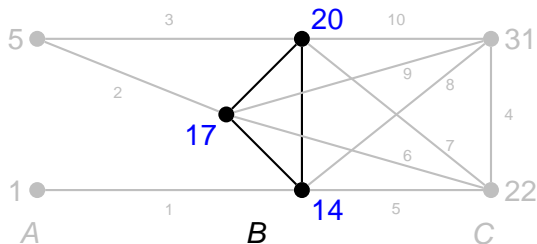


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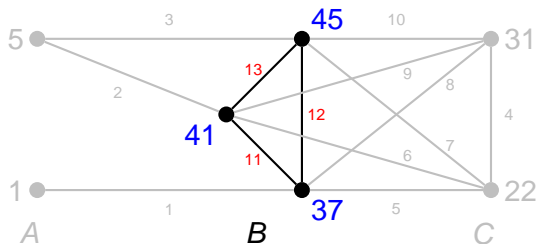


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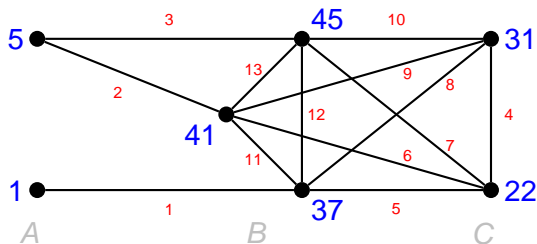


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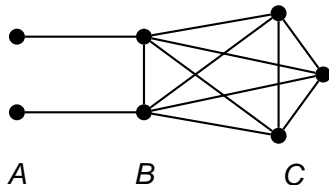


Which graphs are these?

Proposition

The following are equivalent for a graph G .

- (i) G has a clique B such that each vertex in G is adjacent either to all of B or to nothing outside B .*
- (ii) G is split or canonically decomposable.*

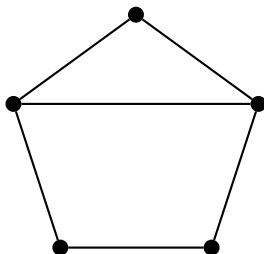


For the future

Prove the antimagic labeling conjecture for graphs which are indecomposable and not split.

Most (interesting) graphs *are* indecomposable and not split.

- Degree sequence characterization [Tyshkevich, 1980]
- A_4 -structure characterization [B, West]: graph is “connected” by its induced $2K_2$'s, P_4 's, and C_4 's.

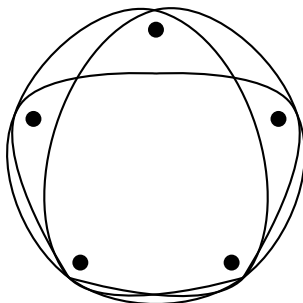


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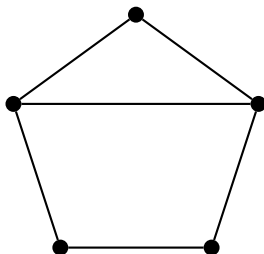


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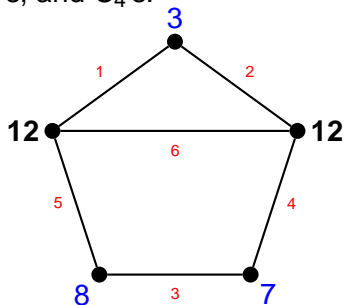


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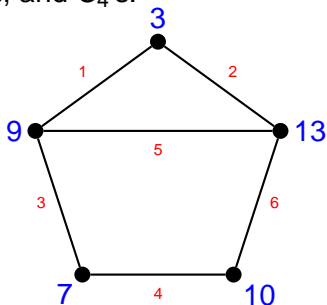


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