The degree-associated reconstruction number of a graph

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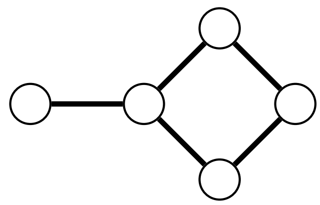
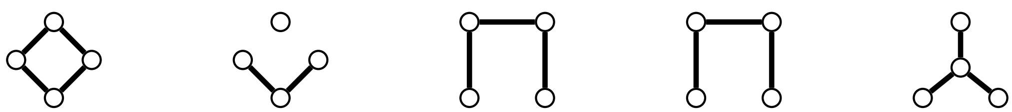
Joint work with D. B. West

Graph reconstruction



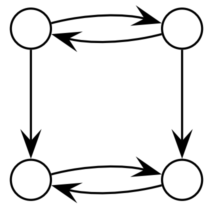
Cards in the deck:



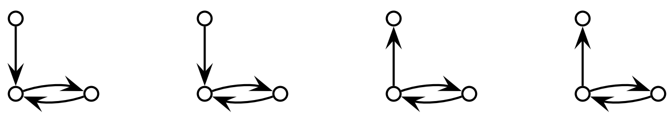
Reconstruction Conjecture: No two nonisomorphic graphs on ≥ 3 vertices have the same deck.

$$\{\circ \quad , \circ\}$$

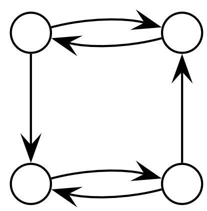
Digraph reconstruction



Deck:



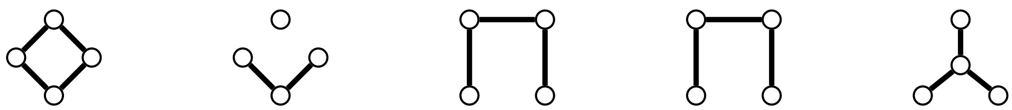
Digraph Reconstruction Conjecture: No two nonisomorphic digraphs (with enough vertices) have the same deck.



Theorem. [Stockmeyer, 1977, 1981, 1988] There are infinitely many non-reconstructible digraphs.

One difference

Graphs

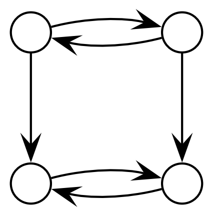


3, 2, 2, 2, 1

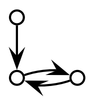
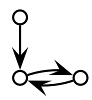
Digraphs

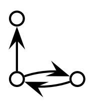
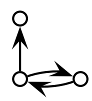
$(2,1)$, $(2,1)$, $(1,2)$, $(1,2)$

Degree-associated reconstruction



Dacards in the dadeck:

$((2,1),$  $)$ $((2,1),$  $)$

$((1,2),$  $)$ $((2,1),$  $)$

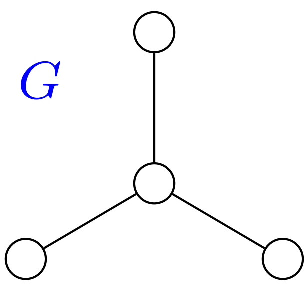
Conjecture: No two nonisomorphic (di)graphs have the same dadeck.

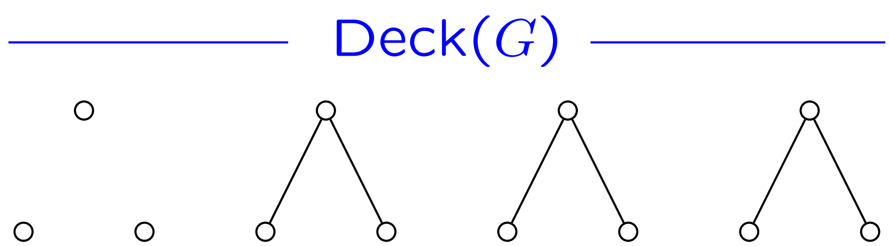
 

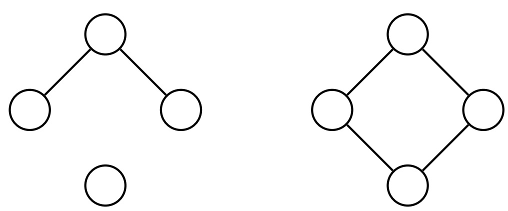
$\{(1, \circ), (1, \circ)\}$ $\{(0, \circ), (0, \circ)\}$

Reconstruction number
(Harary, Plantholt 1985)

rn(G) = size of smallest collection of cards which determines G.







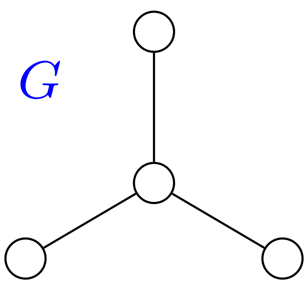
rn(G) $= 3$

Degree-associated reconstruction number

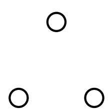
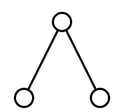
(Ramachandran 2000)

drn(G) = size of smallest collection of dacards which determines G.

G



Dadeck(G)

(3, ), 3 · (1, )

drn(G) = 1

Some results

$$\text{drn}(G) \leq \text{rn}(G)$$

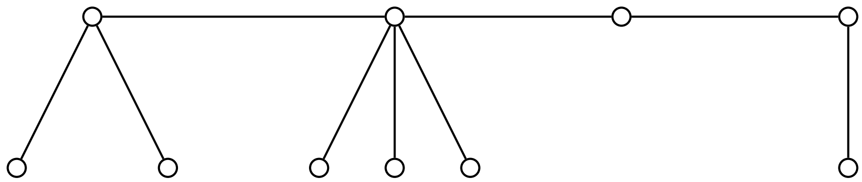
Graph	rn(G)	drn(G)
C_n $(n \geq 5)$	3	3
K_n	3	1
$K_{m,n}$ $(2 \leq m < n)$	3	2
tK_n $(t > 1, n \geq 2)$	$n+2$	3
Almost every graph	3	2
Trees	3	?
Vertex-transitive	?	?

Trees

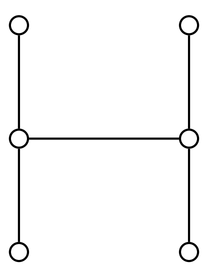
Theorem. [Myrvold, 1990] For any tree T on $n \geq 5$ vertices, $\text{rn}(T) = 3$.

Proposition. For any tree T on $n \geq 3$ vertices, $\text{drn}(T) \leq 3$, with $\text{drn}(T) = 1$ iff T is a star.

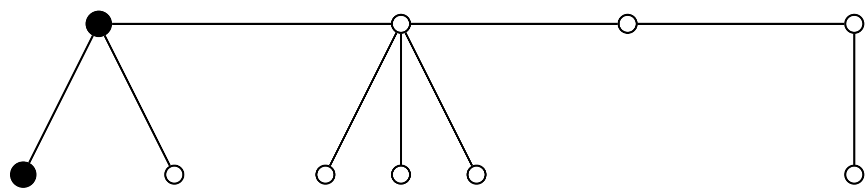
Caterpillars

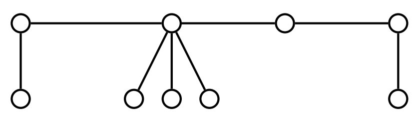


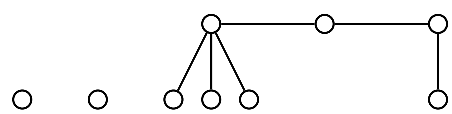
Theorem. [B, West] With the exception of stars and the graph shown below, every caterpillar C satisfies $\mathrm{drn}(C) = 2$.



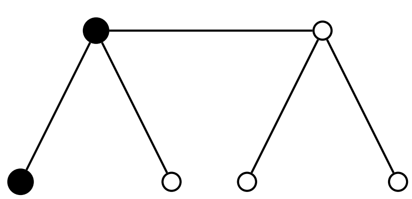
Caterpillars

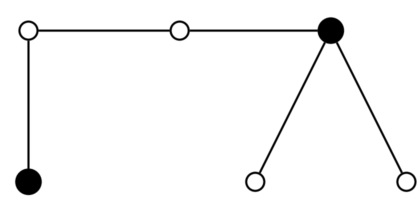


$(1,$  $)$

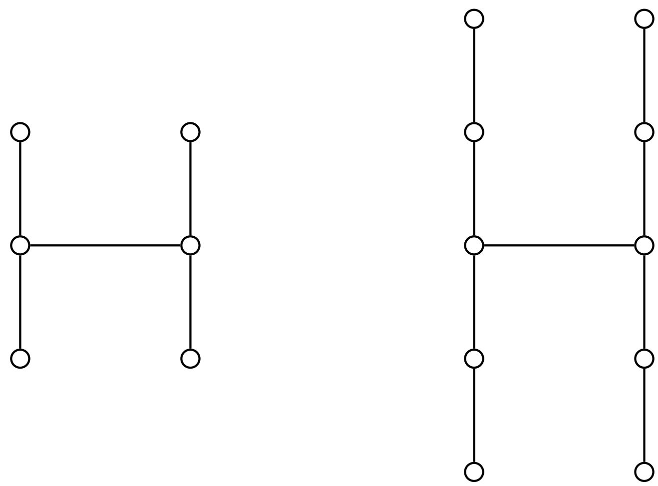
$(a,$  $)$

Doesn't always work:





Trees



H_1 H_2

$$\text{drn}(H_1) = \text{drn}(H_2) = 3$$

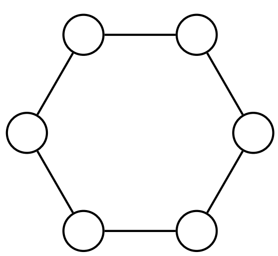
$$\text{drn}(H_k) = 2 \text{ for } k \geq 3$$

Question: Are there infinitely many trees T for which $\text{drn}(T) = 3$?

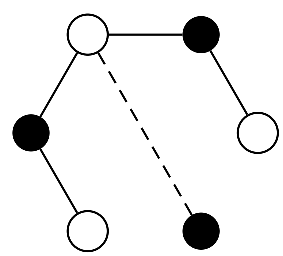
Regular graphs

Proposition. [B, West] If G is r-regular, then $\text{drn}(G) \leq \min\{r+2, n-3-r\}$.

$\Delta(\text{card}) \leq r$.



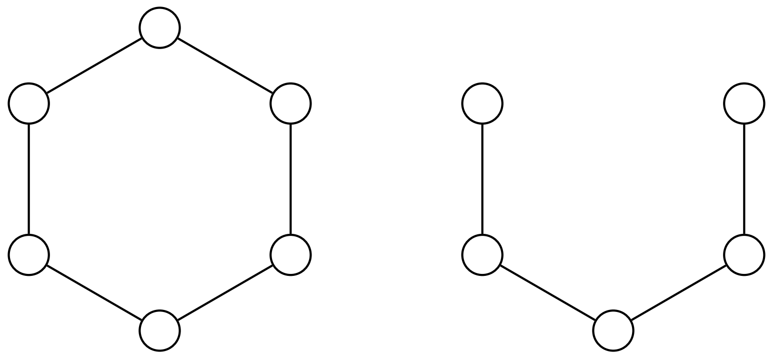
G



H

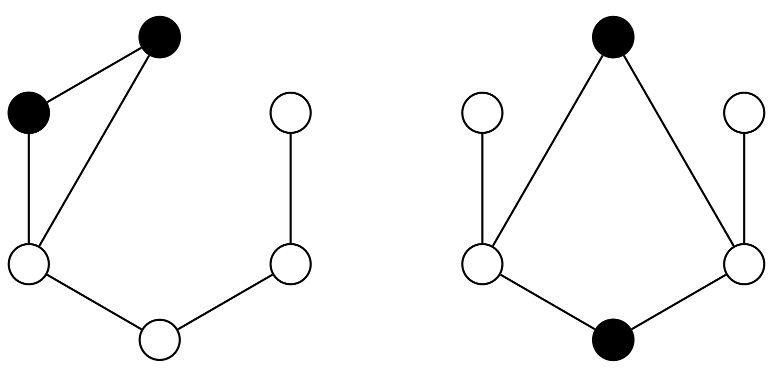
Sharp for $tK_{m,m}$ where $t, m > 1$.

Vertex-transitive graphs



All dacards are the same . . . so higher drn(?).

Proposition. [B, West] If G is vertex-transitive but not complete or edgeless, then $\text{drn}(G) \geq 3$.



Vertex-transitive graphs

Theorem. [Ramachandran, 2000, 2006]

- drn(K_n) $= 1$ for all $n \geq 1$;
- drn(C_n) $= 3$ for $n \geq 4$;
- drn($K_{m,m}$) $= 3$ for $m \geq 2$;
- drn(tK_n) $= 3$ for $t, n \geq 2$;
- drn($tK_{m,m}$) $= m + 2$ for $t, m \geq 2$.

Theorem. [B, West]

- drn(Q_n) $= 3$ for all $n \geq 2$;
- drn($K_n \Box K_2$) $= 3$ for $n \geq 2$.

Question: Given $k \in \mathbb{N}$, are there infinitely many (vertex-transitive) graphs G with drn(G) $=$ k? Connected and co-connected (vertex-transitive) graphs?

Further questions

- Conjecture (Manvel, 1988): A digraph is reconstructible from dacards if the underlying graph is reconstructible.

- Conjecture: All (di)graphs are reconstructible from their dadecks.