Conditional Probability

1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and $P(B|A) = \frac{P(A \cap B)}{P(A)}$.
2. $P(\overline{A}|B) = 1 - P(A|B)$.
3. If A, B independent, then $P(A \cap B) = P(A)P(B)$ and so $P(A|B) = P(A)$ and $P(B|A) = P(B)$.
4. If A, B independent, then$P(A|C)P(B|C) = P[(A \cap B)|C]$.

Bayes' Theorem

1. If $B_1, B_2, \ldots, B_n$ form a partition of the sample space (ie. if exactly one of the B_i occur), then for $j = 1, 2, \ldots, n$,

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$$

   Note that the denominator is $P(A)$.

2. This formula is very important for the exam. Know it well!

After tonight you should be able to do #1-17, 19-27.
Questions For The Lecture

1. (# 6) A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease. Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

   a) 0.115 b) 0.173 c) 0.224 d) 0.327 e) 0.514

2. (# 9) An insurance company examines its pool of auto insurance customers and gathers the following information:

   (a) All customers insure at least one car.
   (b) 70% of the customers insure more than one car.
   (c) 20% of the customers insure a sports car.
   (d) Of those customers who insure more than one car, 15% insure a sports car.

   Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

   a) 0.13 b) 0.21 c) 0.24 d) 0.25 e) 0.30

3. (# 25) A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

   a) 0.324 b) 0.657 c) 0.945 d) 0.950 e) 0.995

4. (# 26) The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker?

   a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{2}$ e) $\frac{2}{3}$

Quiz

1. (# 13) An actuary is studying the prevalence of three health risk factors, denoted A, B, and C, whithin a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B is $\frac{1}{3}$. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

   a) 0.280 b) 0.311 c) 0.467 d) 0.484 e) 0.700

2. (#27) A study of automobile accidents produced the following data:

Model Year	1997	1998	1999	Other
Proportion of all vehicles	0.16	0.18	0.20	0.46
Probability of involvement in an accident	0.05	0.02	0.03	0.04

   An automobile from one of the model years 1997, 1998, and 1999 was involved in an accident. Determine the probability that the model year of this automobile is 1997.

   a) 0.22 b) 0.30 c) 0.33 d) 0.45 0.50