MATH 124 REVIEW SOLUTIONS

Problem 1.

(a)

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Problem 2.

(a) $\# \text{ of distinct words} = \dfrac{\# \text{ total words}}{\# \text{ repeats!}} = \dfrac{8!}{3!} = 8 * 7 * 6 * 5 * 4$

(b) Count the number of distinct four letter words that contain no O, one O, two O's, and three O's. Then the total possible distinct four letter words is given by $P(5,4)C(4,0) + P(5,3)C(4,1) + P(5,2)C(4,2) + P(5,1)C(4,3)$

Problem 3.

(a) $Pr(\text{Not getting a sum of 4}) = 1 - Pr(\text{getting a sum of 4}) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12}$ since we can get a sum of 4 in exactly three ways: $(1,3), (3,1)$, or $(2,2)$.

(b) There is one way to get all heads and one way to get all tails of $2^3 = 8$ possible outcomes, so $Pr(\text{all heads or all tails}) = \frac{2}{8} = \frac{1}{4}$.

Problem 4.

Probability Distribution:

outcomes	*probabilities*
\$3	2/6
\$1	4/6

So the expected value is $\$3 * \frac{2}{6} + \$1 * \frac{4}{6} = \$\frac{10}{6}$.

Problem 5.

$Pr(\text{sum of 8}|\text{at least one 3}) = \dfrac{Pr(\text{sum of 8 and at least one three})}{Pr(\text{at least one three})} = \dfrac{2}{11}$. Note that we have 11 in the denominator because we do not count $(3,3)$ twice and 2 in the numerator since the only possibilities are $(3,5)$ and $(5,3)$.

Problem 6.

$n = 8$; $p = 4/10$ so $q = 6/10$. Recall that $b(k) = C(n,k)p^k q^{n-k}$ is the probability of exactly k successes.

(a) $\text{Pr(exactly two successes)} = b(2) = C(8,2)(0.4)^2(0.6)^6 = \frac{8*7}{2}(0.4)^2(0.6)^6$

(b) $\text{Pr(at most two successes)} = B(2) = b(0)+b(1)+b(2) = C(8,0)(0.4)^0(0.6)^8 + C(8,1)(0.4)^1(0.6)^7 + C(8,2)(0.4)^2(0.6)^6 = (0.4)^0(0.6)^8 + 8(0.4)^1(0.6)^7 + \frac{8*7}{2}(0.4)^2(0.6)^6$

(c) $\text{Pr(at least two successes)} = 1 - B(1) = 1 - [b(0) + b(1)] = 1 - C(8,0)(0.4)^0(0.6)^8 - C(8,1)(0.4)^1(0.6)^7 = 1 - (0.4)^0(0.6)^8 - 8(0.4)^1(0.6)^7$.

Problem 7.
$x_1 = 14; x_2 = -22$

Problem 8.
Row reduce to $\left[\begin{array}{ccc|c} 1 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & k+1 \end{array}\right]$ and conclude that the system is consistent if and only if $k+1=0$, so $k=-1$.

Problem 9.
The three lines define a triangular region of the plane. Find the corner points by solving the systems of equations in pairs.

corner points	(*a*) z = -2x + 3y	(*b*) z = 5x - 2y
$(2/3, 5/3)$	$11/3$	0
$(5/3, 8/3)$	$14/3$	3
$(20/3, -4/3)$	$-52/3$	$108/3$

So for part (a) we have a minimum of $-52/3$ and for part (b) we have a maximum of $108/3 = 36$.

Problem 10.
Order does not matter so # of ways $= C(10,6) * C(7,3) = 210 * 35$.

Problem 11.
Order *does* matter so # of ways $= P(15,6) = 15 * 14 * 13 * 12 * 11 * 10$.

Problem 12.
Calculate the reduced row echelon form of the augmented matrix: $\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 15 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$.
Since the second column does not have a leading one, y is the parameter. Call it t. Then the parameterized solution is $x = 15 - 4t$, $y = t$, $z = -3$ and $w = -2$.

Problem 13.
$Pr(W|J) = \frac{Pr(W \cap J)}{Pr(J)} = \frac{0.7*0.0032}{0.7*0.0032+0.3*0.0061}$.

Problem 14.

(a) 7^3
(b) $7 * 6 * 5$

Problem 15.
This is a Bernoulli process with $p = 1/6$, $q = 5/6$, and $n = 4$. The probability that exactly 3 fives occur is $b(3) = C(4,3)(1/6)^3(5/6) = \frac{20}{6^4}$.

Problem 16.
$$\begin{bmatrix} -2 & -11 & 2 & -5 \\ -1 & 12 & 7 & -12 \end{bmatrix}$$

Problem 17.
Probability Distribution:

outcomes	*probabilities*
1	1/8
2	5/8
3	1/8
4	1/8

Note that you get a 2 if you draw an odd number or if you draw a four. The expected value is $1 * 1/8 + 2 * 5/8 + 3 * 1/8 + 4 * 1/8 = 18/8$.

Problem 18.
Simultaneously row reduce to obtain

$$\left[\begin{array}{ccc|cc} 1 & 2 & -3 & -1 & 2 \\ 3 & 5 & 3 & 2 & -3 \\ -5 & -9 & 8 & 5 & -16 \end{array}\right] \rightarrow \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -12 & 47 \\ 0 & 1 & 0 & 7 & -27 \\ 0 & 0 & 1 & 1 & -3 \end{array}\right]$$

Problem 20 $(a) A \cup B = \{x, y, s, t, 1, 2\}$
$(b) A \cap B' = \{x, y\}$

Problem 21 (a) $515 - 215 = 300$. (b) $450 - 215 = 235$. (a)' $\frac{515-215}{960} = \frac{5}{16}$. (b)' $\frac{450-215}{960} = \frac{235}{960}$.

Problem 22 $Pr(2) = \frac{1-\frac{1}{3}-\frac{1}{9}}{4} = \frac{5}{36}$; $Pr(E) = Pr(2) + Pr(4) + Pr(6) = \frac{5}{36} + \frac{5}{36} + \frac{1}{9} = \frac{7}{18}$.

Problem 23 (a) $Pr(E) = 1/2, Pr(G) = 1/12, Pr(E \cap G) = 1/18$. $Pr(E)Pr(G) \neq Pr(E \cap G)$. So E and G are not independent.
(b) $Pr(E|G) = \frac{Pr(E \cap G)}{Pr(G)} = \frac{\frac{1}{18}}{\frac{1}{12}} = \frac{2}{3}$.

Problem 24 Note, for two lines with slope k_1, k_2, they are parallel if and only if $k_1 = k_2$; they are perpendicular if and only if $k_1 \cdot k_2 = -1$. The slope of $4x - 3y = 5$ is $k = 4/3$. So a line parallel to it must have the equation $y = \frac{4}{3}x + b$ with b to be determined. But from the assumption this line passes $(5, -1)$, so $-1 = \frac{4}{3}5 + b$. Thus $b = \frac{-23}{3}$. And the equation should be

$$y = \frac{4}{3}x + \frac{-23}{3}.$$

The slope of the line perpendicular to $4x - 3y = 5$ should be $\frac{-3}{4}$. It then have the equation $y = \frac{-3}{4}x + b$ with b such that $(6, 2)$ satisfy the equation, i.e

$$2 = \frac{-3}{4}6 + b.$$

Then $b = \frac{13}{2}$. And the equation is

$$y = \frac{-3}{4}x + \frac{13}{2}.$$

Problem 25 (**I changed the number 12oz to 20oz in this problem to easy the calculation**)

Let x be the number of softballs they should make and y be the number of hard balls they should make.

$$100x + 30y = 3000$$
$$20x + 8y = 720;$$
$$x \geq 0, y \geq 0$$

Corner points: $(12, 60), (0, 90), (30, 0), (0, 0)$.

Problem 26 $AC = \begin{bmatrix} -6 & 46 \\ 9 & -12 \end{bmatrix}$; $BC = \begin{bmatrix} 18 & -21 \\ 16 & -7 \end{bmatrix}$.