

MATH 444 section E13, Exam #2

Name

1. (14pts)

Find the following limits: (no procedure needed)

(a) $\lim_{x \rightarrow 2} \frac{x}{1+x} - x^2$.

(b) $\lim_{x \rightarrow 1} \frac{\sqrt{1+8x} - \sqrt{1+3x}}{1+2x^2}$.

(c) $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$.

Find the derivative of the following function: (no procedure needed)

(d) $f(x) = \frac{x}{1-x^2}, x \neq \pm 1$.

(e) $f(x) = (\sin x^2)^3$.

(f) $f(x) = \tan x^3, -1 < x < 1$.

(g) $f(x) = \arccos x, -1 < x < 1$.

2. (18 pts) Consider $f(x) = 2|x|$, $x \in \mathbb{R}$

(i) Use definition to prove that f is continuous on \mathbb{R} .

(ii) Use definition to find the derivative of $f(x)$ for $x \neq 0$.

(iii) Prove that f is not differentiable at $x = 0$.

3. (18 pts) Let $f(x) = \sin(\frac{1}{x})$ for $x > 0$.

(i)(3pts) We know $\sin x, x$ is continuous for $x \in \mathbb{R}$. Is $f(x)$ continuous on $(0, \infty)$? Why?

(ii)(3pts) Given $\sin(\frac{\pi}{4}) = \sin(\frac{3\pi}{4}) = 0.707$, $\sin \frac{\pi}{2} = 1$, prove that there exist at least two x , $\frac{4}{\pi} < x < \frac{4}{3\pi}$ such that $f(x) = 0.8$ without using arcsin function.

(iii)(6pts) Prove that $f(x)$ is not uniformly continuous on $(0, \frac{1}{\pi})$.

(iv) (3pts) Let g be a continuous function defined on $[-1, 1]$, $g(x) > 0$. Is the composition $g \circ f$ uniformly continuous on $[\frac{1}{\pi}, \frac{2}{\pi}]$? Why?

(v) (3pts) Prove that there exist a positive number α such that $g \circ f(x) > \alpha$ for all $x > 0$.

Bonus questions:

(i) (2pts) Show that if f is continuous on $(0, 1)$ then f^2 is uniformly continuous on $(0, 1)$ by using the properties of continuous functions from Chapter 5.

(ii) (2pts) Use the knowledge we learned recently to prove that $\sin^2 x + \cos^2 x = 1$ for all $x \in \mathbb{R}$ provided $\sin^2 0 + \cos^2 0 = 1$.