

MATH 444 section E13, QUIZ #2

Name

1. (10 pts) Use the definition of the limit of a sequence to prove that

$$\lim\left(\frac{3n+5}{2n+4}\right) = \frac{3}{2}.$$

Solution: $\forall \epsilon > 0$, Let K_ϵ be a natural number bigger than $\frac{1}{2\epsilon}$ (by Archimedean property 2.4.3, such a K_ϵ exists). Then

$$\left|x_n - \frac{3}{2}\right| = \left|\frac{3n+5}{2n+4} - \frac{3}{2}\right| = \left|-\frac{1}{2n+4}\right| < \frac{1}{2n} < \frac{1}{\frac{1}{\epsilon}} = \epsilon \quad (*)$$

for all $n \geq K_\epsilon$. By definition, we conclude $\lim\left(\frac{3n+5}{2n+4}\right) = \frac{3}{2}$. ■

(On the scratch paper, you can work on (*) first to figure out how to choose K_ϵ .)

2. (10 pts) Determine the following sequences (x_n divergent or convergent, find their limit if they are convergent. No procedure needed.

(i) $x_n = 2^n$.

Divergent. (Because it is unbounded)

(ii) $x_n = \sin \frac{n\pi}{2}$.

Divergent. (We did it in class that it has subsequences converge to different numbers)

(iii) $x_n = \frac{n-1}{n+1}$.

Converges to 1.

(iv) $x_1 = 2$ and $x_{n+1} = 2 - \frac{1}{x_n}$ for all $n \in \mathbb{N}$.

Converges to 1 (It converges because it is bounded and monotone which you can verify on the scratch paper. Then taking limit on both sides on $x_{n+1} = 2 - \frac{1}{x_n}$ and you can find the limit by solve the equation $x = 2 - 1/x$.)