MATH 286 section G1, QUIZ #2

Names........Answer........

1 (15 pts) Solve the following differential equations.

(i)$y'' + 5y' + 6y = 0$ with $y(0) = 1, y'(0) = 3$.

Solution: $r^2 + 5r + 6r = 0$

$$(r+2)(r+3) = 0$$

$$r = -2, -3$$

$$y = C_1 e^{-2x} + C_2 e^{-3x}$$

$y(0) = 1$ so $C_1 + C_2 = 1$

$y'(0) = 3$ so $-2C_1 - 3C_2 = 3$

Then $C_1 = 6$, $C_2 = -5$

$$\boxed{y = 6e^{-2x} - 5e^{-3x}}$$

(ii)$y^{(3)} - y'' + 2y' - 2y = 0$.

Solution: $r^3 - r^2 + 2r - 2 = 0$

$$r^2(r-1) + 2(r-1) = 0$$

$$(r^2+2)(r-1) = 0$$

$$r = 1, \pm\sqrt{2}i$$

$$y = C_1 e^x + C_2 \cos(\sqrt{2}x) + C_3 \sin(\sqrt{2}x)$$

(iii)$y^{(4)} + 2y'' + y = 0.$

$$r^4 + 2r^2 + 1 = 0$$

$$(r^2+1)^2 = 0$$

$r = \pm i$ with 2-multiplicity

$$y = C_1 \cos x + C_2 \sin x + C_3 x \cos x + C_4 x \sin x.$$

2 (5pts) Given $y_1 = 1, y_2 = x^2, y_3 = x^2 \ln x$ as three solutions of

$$y^{(3)} - \frac{1}{x}y'' + \frac{1}{x^2}y' = 0,$$

for $x \in I = (0, \infty)$. Verify that y_1, y_2, y_3 are linearly independent on I by their Wronskian. Find all solutions of the differential equation.

$$W(y_1, y_2, y_3) = \begin{vmatrix} 1 & x^2 & x^2 \ln x \\ 0 & 2x & 2x\ln x + x \\ 0 & 2 & 2\ln x + 3 \end{vmatrix}$$

At $x = 1 \in (0, \infty)$. $W(y_1, y_2, y_3) = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}$

$$= 4 \neq 0$$

So $W(y_1, y_2, y_3)$ is not always 0 on I so y_1, y_2, y_3 linearly independent.

Therefore, $y = C_1 + C_2 x^2 + C_3 x^2 \ln x.$