MATH 286 section G1, QUIZ #1

Names........Answer........

1 (5 pts) Find **all** solutions of

$$\frac{dy}{dx} = (y-2)^3 x.$$

Solution: If $y \neq 2$

$$\frac{1}{(y-2)^3}\,dy = x\,dx$$

$$\int \frac{1}{(y-2)^3}\,dy = \int x\,dx + C_1$$

$$-\frac{1}{2}(y-2)^{-2} = \frac{x^2}{2} + C_1$$

$$(y-2)^{-2} = -x^2 + C_2$$

$$y - 2 = \pm\sqrt{\frac{1}{C_2 - x^2}}$$

With any constant $C_2 > 0$ for $x \in (-C_2, C_2)$.

Singular solution: $y \equiv 2$

2 (5pts) Find implicit solutions of

$$(y^2 + 2ye^x + 6e^{3y}\sin 2x)dy + (y^2e^x + 4e^{3y}\cos 2x + 2x)dx = 0.$$

Solution: Implicit solutions $F(x,y) \equiv C_1$

$$F(x,y) = \int y^2 + 2ye^x + 6e^{3y}\sin 2x \, dy + g(x)$$

$$= \frac{y^3}{3} + y^2e^x + 2e^{3y}\sin 2x + g(x)$$

$$\frac{\partial F}{dx} = y^2e^x + 4e^{3y}\cos 2x + g'(x)$$

so $g'(x) = \underline{x^2} + C$

Therefore $F(x,y) = \frac{y^3}{3} + y^2e^x + 2e^{3y}\sin 2x + x^2 + C_2$

The implicit solutions are $\frac{y^3}{3} + y^2e^x + 2e^{3y}\sin 2x + x^2 \equiv C$

3 (10pts) Find a particular solution satisfies

$$x\frac{dy}{dx} + 6y = 3xy^{\frac{4}{3}}$$

for all $x > 0$ with the initial condition $y(1) = 8$.

Solution: Let $v = y^{1-\frac{4}{3}} = y^{-\frac{1}{3}}$

From original equation, we get:

$$\cancel{x}\frac{dy}{dx} + 6\frac{y}{x} = 3y^{\frac{4}{3}}$$

$$y^{-\frac{4}{3}}\frac{dy}{dx} + 6y^{-\frac{1}{3}}\frac{1}{x} = 3$$

That is $$-3\frac{dv}{dx} + 6v\frac{1}{x} = 3$$

$$\frac{dv}{dx} - 2\frac{v}{x}\cancel{x} = -1$$

$$x^{-2}\frac{dv}{dx} - 2vx^{-3} = -x^{-2}$$

$$\frac{d\,vx^{-2}}{dx} = -x^{-2}$$

$$vx^{-2} = -\int x^{-2} + C$$

$$= +x^{-1} + C$$

$$v = +x + Cx^2$$

$$y^{-\frac{1}{3}} = Cx^2 + x$$

$$y = (Cx^2 + x)^{-3}$$

We go back to check the singular solution $y \equiv 0$. It does not satisfies $y(1) = 8$ so we exclude it.

Since $y(1) = 8$ we get

$$(C+1)^{-3} = 8$$

$$C + 1 = \frac{1}{2}$$

$$C = \frac{-1}{2}$$

$$y = \left(\frac{-1}{2}x^2 + x\right)^{-3}$$