

Some problems of Exam2 from Previous Semester

1. Let

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}.$$

- (1) (8 points) Compute the determinant of A .
- (2) (10 points) Is A invertible? Why?
- (3) (10 points) Compute the determinant of AA^T .
- (4) (10 points) Does the equation $A\mathbf{x} = \begin{bmatrix} 27 \\ 101 \\ 34 \end{bmatrix}$ have a solution? Why?
- (5) (6 points) Does $Ax = b$ have a **unique** solution for any b ? Explain.
- (6) (6 points) Is \mathbb{R}^3 the Span of columns of A ? Explain .
- (7) (10 points) Find the inverse of A by using the adjugate matrix.
- (8) Find the solution of the system $Ax = b$ for $b = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

2. (15 points) Mark each statement True or False and justify your answer.

(1) A vector is an arrow in 3-dimensional space.

(2) A subset H of a vector space V is a subspace of V if the zero vector is in H .

(3) A subspace is also a vector space.

(4) Every subspace of \mathbb{R}^3 has the form $Span\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for some vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ in \mathbb{R}^3 .

(5) If f is a function in the vector space V of all real valued functions on \mathbb{R} and if $f(t) = 0$ for $t = 0$, then f is the zero vector in V .

3. (10 points) Consider the following subset W of \mathbb{R}^3 :

$$W = \left\{ \left[\begin{array}{c} c - 6d \\ 5d \\ 3c + 10d - 1 \end{array} \right] : c, d \text{ real} \right\}.$$

Is W a subspace of \mathbb{R}^3 ? If, yes, find a set S that spans W .

4. (5 points) Let

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}.$$

Draw a parallelogram in \mathbb{R}^2 whose area equals the determinant of A .