Solution of 1.3.9.

S and T are denumerable, so $S \cup T$ won't be finite. Therefore, to prove $S \cup T$ is denumerable, we only need to show $S \cup T$ is countable.

Since S and T are denumerable, we have bijections f and g such that $f: \mathbb{N} \to S$, $g: \mathbb{N} \to T$ are bijections. Define $h: \mathbb{N} \to S \cup T$ as follows.

$$h(2n-1) = f(n) \quad \text{for any } n \in \mathbb{N}$$

$$h(2m) = g(m) \quad \text{for any } m \in \mathbb{N}.$$

Then it's easy to see that h is a surjection from $\mathbb{N}$ onto $S \cup T$. By Theorem 1.3.10, $S \cup T$ is a countable set. Since it's infinite, $S \cup T$ is denumerable.

Here is why h is a surjection. For any $x \in S \cup T$,

if $x \in S$ then $\exists\, n_x \in \mathbb{N}$ s.t. $f(n_x) = x$

so $\exists\, 2n_x - 1 \in \mathbb{N}$ s.t. $h(2n_x - 1) = x$

if $x \in T$ then $\exists\, m_x \in \mathbb{N}$ s.t. $g(m_x) = x$

so we find $2m_x \in \mathbb{N}$ s.t. $h(2m_x) = x$

Therefore h is a surjection