

MATH 385, Section D2, Midterm Exam 1

Name .....; ID #.....

1. (18pts) In this problem, you need to mark in the "[ ]" in front of each option as either T(=True) or F(=false). No explanations of your answers are necessary.

The differential equation  $dy/dx = xy^3 - xy$

- [ ] (a) is a first order equation;
  - [ ] (b) is a separable equation;
  - [ ] (c) is a Bernoulli equation;
  - [ ] (d) is a linear first order equation;
  - [ ] (e) is a third order equation;
  - [ ] (f) has one and only one solution for the initial condition  $y(0) = 1$ .
2. (10 pts) Solve the differential equation  $dy/dx = \cos 3x + e^{2x}$  for the initial condition  $y(0) = 2$ .

3. (16 pts) Find the general solution and any singular solution of the differential equation  $dy/dx = y(y - 1)$ .

4. (20 pts) A tank initially contains 120 gal of pure water. Brine containing 1 lb of salt per gallon enters the tank at 3 gal/min, and the (perfectly mixed ) solution leaves the tank at 4 gal/min; thus the tank is empty after 2 hours.

(a) Prove that the amount  $x$  of salt in tank satisfy the differential equation

$$\frac{dx}{dt} = 3 - \frac{4x}{120 - t}, \quad x(0) = 0.$$

(b) Find the amount of salt in tank after 1 hour by solving the differential equation in (a).

5. (36 pts) Find general solutions of the following differentail equations. If an initial condition is given, find the corresponding particular solution. Throughout, primes denote derivatives with respect to  $x$ .

(i)  $y' = (3x + 2y)^2$ ;

$$(ii) y' = \frac{x+2y}{y-2x}$$

$$(iii) 2xy' + y^3 e^{-2x} = 2xy$$